Richard Price and the
Origins of Bayesian Decision Theory

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Abstract

This paper examines the earliest known contribution to Bayesian decision theory: the reply that Richard Price (1723-91) made to David Hume's sceptical argument against Christian miracles. Contrary to modern presentations, Price's primary application of Bayes's theorem is shown to be distinct from the conventional Bayesian approach to the interpretation of testimony. The 'rational intuition' used to motivate Price's prior distribution is compared with modern intuitionism and substantive differences identified. In conclusion, the theological foundation of Price's Bayesian view on Christian miracles is found to be consistent with empirical observation because, being a dissenting minister, Price was able to avoid doctrinaire claims regarding miracles. This interpretation permits essential issues in the debate with Hume to be properly situated within the broader philosophical and theological debates of those times.

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1. Introduction

The treatment of Richard Price (1723-1791) in the history of economic thought is an enigma. So much of what Price contributed still resonates today, providing insights into subjects ranging from social security reform to the proper management of insurance companies to the political implications of government debt issuance. Yet, compared to the voluminous efforts expended on detailing the contributions of contemporaries in political economy such as Adam Smith or David Hume, Price has received scant attention.[2] Most of the notoriety Price has received in modern venues focuses on his subsequent contributions to the subjects of actuarial science and political theory.[3] The bulk of these contributions came after the appearance of Price (1768) where, in Dissertation IV, Bayes's theorem is applied to the important theological quandary raised by Hume's sceptical attack on Christian miracles (Hume's attack).[4] Recognizing that it was the communication of Bayes's theorem to the Royal Society in 1763 that earned Price election to the Royal Society in 1765, the argument advanced in Dissertation IV is almost certainly the first known contribution to Bayesian decision theory. Though this argument still attracts some modern interest, unanswered questions remain.[5] After providing a brief overview of the contributions of Richard Price that are relevant to economics, the paper explores questions still surrounding the birth of Bayesian decision theory.

2. Who was Richard Price?

Those unfamiliar with the basic aspects of the life and contributions of the Rev. Richard Price will find Pearson (1978, p.370-421) and Ogborn (1962) to have helpful overviews. Other important sources include the memoirs by Morgan (1815) and the modern biography by the philosopher Thomas (1977). By any reasonable standard, it is an understatement to say that Richard Price was

a remarkable individual; a caricature for English intellectual achievement in the 18th century, despite being, first and foremost, a dissenting minister and theologian. Born in 1723, Price was the son, by a second marriage, to Rees Price: "The early influences on him were the severe Calvinism of the father and the gentle character of his beautiful and delightful mother." (Pearson 1978). As it turns out, being raised and educated as a religious dissenter outside the educational institutions of the Anglican church was fortuitous for Price. At a relatively young age, Price was obliged to make his own way in the world and decided he would seek the help of his uncle in London. It was in London, studying at the dissenting academy founded by one Mr Coward, Price had the good fortune to be instructed by John Eames (1685-1744), FRS, a capable mathematician and friend of Isaac Newton. "Thus it came about that Richard got an education of university standard, especially in mathematics and theology, possibly a better education than he would have received at the universities" (Pearson 1978).

From the time of his graduation from the academy in 1744 until 1756, Price served as a family chaplin. During this time he was able to produce an important contribution to moral philosophy, Price (1758), a companion piece for the important works of David Hume that gathered increasing prominence during this period, especially Hume (1739-1740).[6] In 1756, Price came into a substantial legacy and, with the improvement in fortune, was able to get married and dedicate even more time to studies. One of the scholarly tasks that Price undertook was to edit and prepare for submission to the Royal Society, the papers of Thomas Bayes, FRS. Like Price, Bayes was a dissenting minister, and a more than competent mathematician. In a letter dated Nov. 10, 1763, Price communicated to the Royal Society the contents of a theorem unearthed in Bayes's papers. That letter contained the now famous Bayes's Theorem. From this, formal academic recognition came to Price in 1765 with

election to the Royal Society. The degree of Doctor of Divinity was conferred by Marischal College, Aberdeen, in 1767. All of this had been achieved prior to Price's involvement with the Equitable Society, which started in 1768. Perhaps his most important intellectual contribution, Price (1771), was still to come, as were most important political and social contributions, such as Price's moral support and intellectual participation on the American side in the American Revolution.[7]

In the history of actuarial science, September 7, 1762 is significant because the deed of settlement was entered on that date and the Equitable Life Assurance Society commenced operation, e.g., (Lewin 2003). Richard Price was neither a subscriber or a director for the Equitable. Rather, Price served as a consultant on actuarial matters for about 15 years, starting around 1768. The difficulties of commencing the Equitable were considerable. Price was singularly well suited to this task, as evidenced in the theoretical contributions to actuarial science contained in the Price (1771). Benjamin Franklin has been quoted as calling Price (1771), "the foremost production of human understanding that this century has afforded us" (Ogborn 1962, p.92). More recently, Pearson (1978, p.393) refers to the Price (1771) as "perhaps the most remarkable textbook ever issued on actuarial science". The substance of Price's analysis was overwhelming and decidedly unexpected: virtually all the life assurance and benevolent societies of the time, founded with noble motives, faced certain failure to meet future obligations. Within a short period of time, numerous societies either folded or restructured their premium/payout structure to be in conformity with actuarial soundness. It was a natural progression for Price combine contributions from the academic sphere to the practical realm of implementing an actuarially sound life insurance scheme at the Equitable. Yet another of Price's practical contributions was the insistence that life insurance companies be under the direction of a capable mathematician, the actuary.

Social mechanisms aimed at providing for the elderly and infirmed have a long and complicated history, e.g., Lewin (2003). The Church, the extended family, inheritance practices and charitable organizations have all been important vehicles at one time. However, it was not until Bismark's Germany in the 1880s that a state plan with universal coverage was implemented. Yet, Price (1771) proposed an actuarially sound plan to provide state pensions and allowances for the poor who are elderly or sick. This plan was to be a replacement for the inadequate support provided by the Poor Laws. In 1773, the plan was well received by the House of Commons and was passed, only to be rejected by the House of Lords. In 1789, the plan was revived and passed, once again, the House of Commons only to be, once again, rejected by the House of Lords. Price's plan was quite novel. Though the scheme allowed for the use of local funds and local management, the ultimate objective was aimed at universal coverage. In this respect, Price's proposal is of considerable interest. The ensuing attempts to incorporate the plan into legislation justify considering Price to be the 'Father of Old Age Pensions'. That the plan was unsuccessful speaks more of the attitude in the House of Lords, than to the soundness of Price's plan. "It is doubtful whether our gratitude to Price should be more voluble than our execration of the Lords" (Pearson 1978, p.404).

Compared to modern ill-conceived, pay-as-you-go public pension plans, Price's plan was ingenious. As with much of Price's work, it was motivated by moral obligation: despite the substantial amount of money raised by the Poor Laws, "in many places" the poor were "indifferently provided for" (Price 1771, p.313).[8] As a sizeable number of the poor were elderly, Price wanted to apply the principles of actuarial science to providing for the poor. Using this approach, it was apparent that the poor lacked adequate investment vehicles for retirement savings (Price 1771, p.313):

> To make provision for one's old age is so natural a piece of prudence, that it seems at first sight wonderful, that it should not be generally practised by the labouring poor, as it is almost universally by persons in the higher paths of industry: Nor can their negligence in this respect be accounted for, in any other way so naturally, as by ascribing it to their wanting proper opportunities of employing the money they might save, in some safe and easy method that would procure them a suitable advantage from it in the latter periods of their lives.

Price's idea was the establishment of 'Parish Annuities'.

The basic plan underlying the parish annuities was to have the poor provide for their own old age as much as practical. In Price's time, there were few outlets for small savers, in general, and virtually no actuarially sound pension schemes, in particular. These annuities would be administered at the local level, by the Churchwardens and overseers of the parish. The administrators would be responsible for collecting the annuity payments and investing in bank annuities. Any deficiencies in the fund could be made out of the parish poor rates. The fund would make payouts starting at the age of 55 or 60, at the option of the annuitant. The payout would be staggered, for example, £5 for the first five years, £10 for the next five and £15 pounds thereafter (e.g., Price 1771, p.316). To facilitate the scheme, Price provides a table that could be used to determine the appropriate payments to be made under the scheme. It is unfortunate that the House of Lords twice rejected this scheme. Compared to modern universal government pension plans, which are often little more than disguised welfare schemes, Price's parish annuity scheme would have started Old Age pension plans on a sounder footing. Price's plan was designed for actuarial soundness. The plan was voluntary, effectively unbundling the government payments to the aged into a welfare component, provided under the Poor Laws, and a pension component, the parish annuities, which would reward industry and thrift among the poorer classes. Various technical features, such as allowing the option of selecting a specific starting date and the staggering of annuity payments, are incorporated into

modern universal pension plans.

3. The Modern Bayesian Arguments

In theological studies, Price (1768), particularly Dissertation IV, *The Importance of Christianity, the Nature of Historical Evidence and Miracles,* established his "reputation as a theologian with a new view of morals and religion" (Pearson 1978, p.378). Into modern times, Dissertation IV still has relevance due to the seminal application of Bayesian decision theory. There are a number of facets to this contribution. At least since Molina (1940), it has been recognized that the primary mathematical contribution of the 'Bayes's theorem' that Price communicated to the Royal Society was associated with the use of series solutions to arrive at an expression for the ratio of an incomplete to a complete Beta function as the incomplete sum of the binomial $(q + p)^n$. Following Zabell (1989), modern formulation of this uniform prior result focuses on Laplace's rule of succession. It was not Bayes that formulated the modern 'inverse probability' version of Bayes's theorem where non-uniform prior probabilities can be combined with the observed likelihood function to obtain the conditional posterior distribution.[9] In both Laplace's rule of succession and in Price's version of Bayes's theorem, the prior distribution is uniform, the same *a priori* probability is attached to each possible random outcome.[10] The more difficult mathematical problem of combining a non-uniform prior with a particular likelihood is not attempted. It was the uniform prior version of Bayes's theorem that Price used to address a specific logical implication of the sceptical philosophy underpinning Hume's attack.

In the history of probability, references to Price usually focus attention on the connection between Bayes's theorem and Laplace's rule of succession, e.g., Zabell (1988). This not surprising given that Price was the person responsible for introducing the 'proposition' and 'scholium' that have risen

to such prominence in recent times. While the basic problem of using probability theory to infer causes from effects has survived into modern times, e.g., Zabell (2007). The philosophical issues that inspired Bayes to formulate the famous theorem likely originated as a response to Hume (1739-1740) which, among other contributions, contained the sceptical inductive arguments in Hume's attack (Buckle 2007). As Zabell (1989, p.290) observes: "Hume first stated the problem of induction; Bayes first advanced a solution to it. The chronological link between these two events is much closer than is usually recognized." The continued use of the rising sun example in various treatments of the inductive problem crystallizes the historical time line, which includes Hume, Price's appendix to Bayes (1763), and Laplace's "probabilistic solution to the problem of induction" (Zabell 1989, p.292).

Hume's attack continues to attract modern interest due to the fascinating probabilistic complications involved in the argument, e.g., Earman (2000); Hajek (2008). In the process of developing these complications within a modern Bayesian framework, the sceptical element in Hume's attack is restricted somewhat to permit close analysis of the chief proposition: it is never justified to accept second-hand testimony to a miraculous event. This proposition is derived from two basic conditions: miracles, such as a dead man coming back to life, are violations of laws of nature; and, the miraculous events proposed as evidence for the Christian revelation are based on historical second hand testimony.[11] While Hume's attack is structured to encompass all forms of miracles reported by second hand testimony, the strongest logical elements of the attack center on doctrinaire Catholic Christian miracles. This is significant because, being a dissenting Protestant minister, Price did not have to adhere to doctrinaire views on Christian miracles, a faith based theological position that allows, if not requires, miracles to be interventionist and contemporaneously observable violations

of nature. As a consequence, much of the theological debate between Price and Hume is tempered, allowing attention to focus on the probabilistic foundations of Hume's sceptical philosophy. The associated non-Bayesian aspects of the debate over miracles, relevant to understanding the theological underpinnings, have attracted limited modern interest and been largely overlooked.

Expressing Hume's attack in modern Bayesian terms "renders his reasoning perspicuous, and the issue easier to grapple with" (Weintraub 1996, p.360). This requires some definitions. Let: M be the event that a miracle happened and $T[M]$ be the event that testimony to the occurrence of a miracle was given. Letting ~ indicate negation, this produces two 'prior' probabilities which are relevant: $P[M]$ and $P[\sim M]$, the probabilities of a miracle occurring and not occurring. $P[T[M]]$ and $P[\sim T[M]]$, the prior probabilities of testimony to a miracle being given and not being given, are not used directly. Bayesian analysis is about conditional or posterior probabilities, and the most important of these to Hume's attack is: $P[M \mid T[M]]$, the probability that a miracle occurred, given that there was testimony for a miracle. Using the inverse probability form of Bayes's theorem, this conditional probability can be solved as:[12]

$$P[M \mid T[M]] \; = \; \frac{P[M] \; P[T[M] \mid M]}{P[M] \; P[T[M] \mid M] \; + \; P[\sim M] \; P[T[M] \mid \sim M]}$$

where: $P[T[M] \mid M]$ and $P[T[M] \mid \sim M]$ are the conditional probabilities for the truthfulness of the testimony, e.g., $P[T[M] \mid \sim M]$, the probability of testimony to a miracle given that there was no miracle, would be high when liars are the source of testimony.

To evaluate the $P[M \mid T[M]]$ relevant to Hume's attack using this formula, it is necessary to take all relevant prior and conditional probabilities to be strictly positive to avoid indeterminate or trivial solutions. This step requires closer attention. For Hume (1739, p.124): "One wou'd appear

ridiculous, who wou'd say, that 'tis only probable the sun will rise tomorrow, or that all men myst

dye; tho' 'tis plain we have no further assurance of these facts, than what experience affords us".

In referencing the rising sun or the certainty of death "Hume was arguing that there are degrees of

belief which, while not demonstratively certain, exceed all probability" (Zabell 1989, p.293). While

Hume's attack sees a miracle to be much more than a rare occurrence, the modern approach is

consistent with Laplace where $P[M]$ is a remotely small number. In particular, using the rule of

succession solution Laplace provides a best odds estimate of 1,826,214 to 1 for the sun rising

tomorrow based on the length of human experience. Without loss of content, it is sufficient for the

modern argument to take, say, $P[M] = .000001 = 1 - P[\sim M]$. Similarly, regarding second hand

testimony: "The many instances of forged miracles ... begat a suspicion against all relations of this

kind" (Hume 1748, p.119). Hence, let $P[T[M] \mid M] = .5 = P[T[M] \mid \sim M]$. In this case, the

conditional probabilities in the numerator and denominator cancel leaving $P[M \mid T[M]] = .000001$.

Attributing a very high probability weighting of, say, $P[T[M] \mid M] = .999$ to the truthfulness of the

testimony does not change much; this still only gives a conditional probability: $P[M \mid T[M]] = 0$

.0998%. In effect, because a miracle is a virtually impossible event, a violation of the laws of nature,

it is effectively never possible for the reliability of witness testimony to overcome this hurdle. As

Hume (1748, p.115-6) observes: "no testimony is sufficient to establish a miracle unless the

testimony be of such a kind that its falsehood would be more miraculous than the fact which it

endeavors to establish."

Given the role Price played in communicating Bayes's theorem, it is not surprising that attention

given to Dissertation IV has emphasized that part associated with Price's application of Bayes's

theorem to determine the validity of second hand testimony. Yet, it is conventional in modern

studies to situate Hume's attack within the inverse probability framework initially introduced by Laplace and applied to the specific problem of assessing the reliability of testimony. Only a few modern sources recognize the specific details of Price's contribution on the use of prior probabilities.[13] Even those sources that do recognize the contribution: either, fail to examine why Price was concerned with situating probabilities within intervals rather than obtaining specific point estimates; or, fail to recognize that miracles had a different inductive interpretation for Price. As a mathematical contribution, it is the impact of increasing sample size on the length of the confidence interval that is explicitly discussed. In this there is a hint of the problem that motivated the development of sequential analysis in the 20th century, albeit without any recognition of the restrictions imposed by combining a fixed confidence level and interval length, e.g., Klein (2000).

4. Price's Bayesian Argument

Following Buckle (2007, esp. p.xiii-xviii), the general philosophical issues arising in Hume's attack can be traced through the works of Cicero – an essential component of university level education at the time of Hume and Price – to the ancient Greek schools of Stoicism, Epicureanism and Scepticism. Of these, only Stoicism was consistent with the benevolent god of Christian belief, both doctrinaire and dissenting. In contrast, Hume's philosophy can be characterized as "Scepticism with Epicurean sympathies" (Buckle 2007, p.xiv). Elements of Price's Bayesian argument against Hume's attack resemble the ancient Stoics arguing against the Sceptics: "But let them not pretend they are able to prove *a priori*, that no accounts of miracles *can* be true" (Price 1768, p.452). While Stoics and Epicureans made claims to certain knowledge about the gods and the character of human beings, Sceptics maintained that 'no claims to certain knowledge can be justified'. Building on ancient scepticism, Hume presented a philosophy based on empirical observation as the source of

knowledge, but that also admitted, in a fashion, the *a priori* physical laws uncovered in the natural

philosophy detailed in the mathematical and experimental discoveries of Isaac Newton.

Protestant Christian theology evolved considerably during the Enlightenment, providing a

backdrop for the anti-Catholic, British intellectual climate at the time of Hume's attack. Miracles

provided a fundamental demarcation point between Protestant and Catholic theology. In particular,

while "Catholics hold miracles to be a permanent feature of the world ... evidence of ongoing divine

engagement", Protestants conventionally maintained that miracles "occurred only in a past apostolic

age, a special period of divine activity in the world" (Buckle 2007, p.xviii). This allowed Protestants

to pursue arguments for unobservable miracles not available to doctrinaire Christians. In particular,

it is well known that the version of Bayes's theorem presented by Price to the Royal Society assumed

equality of unknown prior probabilities, i.e., a uniform prior distribution. Applied directly to the

Protestant definition of unobservable miracles, uniformity arguably requires $P[M] = P[\sim M]$ and the

prior distributions cancel when solving $P[M \mid T[M]]$ leaving the conditional probability to be

determined by the truthfulness of the testimony which, for Protestant Christian miracles, was

attributed mainly to the Apostles. Price observes (1768, p.447): "Let them shew, that Christ and his

Apostles were either *enthusiasts* or *imposters*, and account for their conduct and writings on one of

those suppositions".

While maintaining $P[M] = P[\sim M]$ could provide sufficient support for miracles among Stoic

dissenting Christians, this does not deal with the sceptical argument advanced by Hume: "no

testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its

falsehood would be more miraculous, than the fact, which it endeavors to establish". Recognizing

that miracles for a dissenting theologian such as Price did not involve contemporaneously observable

violations of nature, it is understandable that Price made a number of arguments implicitly employing a prior belief that $P[M] = P[\sim M]$.[14]   Throughout Dissertation IV, traditional Stoic criticisms of Scepticism are mingled with Bayesian arguments with the overall aim of undermining of Hume's attack. Significantly, the Bayesian arguments are found mostly in a few footnotes in Dissertation IV and follow much the same line as the 'Appendix' to Bayes (1763) written by Price. For example, consider the following demonstration from Dissertation IV that samples of very large size and constancy are required before any validity can be given to a probabilistic statement (Price 1768, p.397):

> If we know no more of an event than that it has happened ten times in eleven trials, and failed once, and we should conclude from hence, that the probability of its happening in a single trial lies between the odds of nine to one and eleven to one, there would be twelve to one *against* being right. – If it has happened a hundred times, and failed ten times, there would be odds of near three to one *against* being right in such a conclusion. – If it happened a thousand times and failed a hundred, there would be an odds *for* being right of a little more than two to one. And supposing the same *ratio* preserved of the number of happenings to the number of failures, and the same guess made, this odds will go on increasing for ever, as the number of trials increases.

The relatively complicated procedure for calculating these specific odds "is proved by mathematical demonstration" (Price 1768, p.411-2) in the Appendix. Unfortunately, the general case deals with events that have both succeeded and failed a number of times and, as a consequence, is not directly applicable to miracles which are more 'unusual'.

   The connection between the Bayesian argument made in Dissertation IV and Bayes's theorem is apparent in the statement of the main problem being examined in Bayes (1763, p.376):

> *Given* the number of times in which an unknown event has happened and failed: *Required* the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named.

This speaks only to the general inductive problem of calculating the degree of probability from a

finite number of trials. The Bayesian intuition that Price extended to Hume's attack is seen by considering the various worked examples given in the Appendix. Following Dale (1991), let x denote the probability associated with the next occurrence of an event S, where S_i is the occurrence of S on trial i, then in modern notation Price initially gives results for:

$$P\left[\frac{1}{2} < x < 1 \mid S_1\right] = \frac{3}{4} \qquad P\left[\frac{1}{2} < x < 1 \mid S_1, S_2\right] = \frac{7}{8}$$

$$P\left[\frac{1}{2} < x < 1 \mid S_1, S_2, S_3\right] = \frac{15}{16}$$

$$P\left[\frac{1}{2} < x < 1 \mid m \; successes \; in \; succession\right] = \frac{2^{m+1} - 1}{2^{m+1}}$$

As Zabell (1989, p.294) observes, though closely connected, this result is not Laplace's rule of succession where the probability of success on the $m + 1$ trial, given that all m trials to that point had been successful, is given as $(m + 1) / (m + 2)$.

   While these types of calculations are supposed to support statements made about sceptical probabilities in Dissertation IV, the implications for Hume's attack are not immediately apparent. Expressed as Laplace's rule of succession, such calculations suggest that the odds for continued successes without failure in a sequence increases with number of individual successes. Because Hume's scepticism permits degrees of belief which, while not demonstratively certain, exceed all probability, as the number of successes increases – i.e., observed instances where rules of nature are not violated – Laplace's rule of succession provides some support for Hume's *a priori* sceptical probability for a miracle. To avoid this, Price uses a Bayesian argument to "mathematically demonstrate" that, even though a particular event has never been seen to occur in ten previous trials,

calculating "the probability of its happening in a single trial [that] lies somewhere between any two degrees of probability that can be named". This produces an unexpected result:

$$P\left[\frac{2}{3} < x < \frac{16}{17} \mid 10 \; successes, \; no \; failures \right] = 0.5013$$

Price (1768, p.397) observes: "The specimen now given is enough to show how very inaccurately we are apt to speak and judge on this subject, previously to calculation."

The Bayesian argument being made by Price concerns the general use of inductive probabilities to support sceptical conclusions, rather than with specific elements of Hume's attack. For this purpose, Price falls back on more traditional theological arguments. Though Hume's attack was particularly sharp, the empirical or materialist Enlightenment debate on miracles had generated intense interest going back at least to Spinoza and Locke. Development of the metaphysical and theological properties of miracles provided by Clarke in the dispute with Liebnitz provided sufficient traditional firepower for Price to counter Hume's attack without employing a Bayesian argument. As such, the text of Dissertation IV is more about the evolution of dissenting Protestant theology on miracles, rather than the first application of Bayesian decision theory to the practical problem of evaluating the credibility of testimony. The Bayesian argument, given largely in lengthy footnotes, was likely added after the bulk of the text of Dissertation IV was completed.

Price recognized that a Bayesian argument could be used to undermine the sceptical claims to *a priori* probability about miracles that appeared in Hume's attack. Though Price was attracted by the logical possibility of refuting Hume's attack mathematically, the actual Bayesian argument presented was decidedly incidental to the overall objective. The heavy lifting in Dissertation IV is achieved by Price's defining a miracle as an unusual event guided by divine intent. Armed with this

definition, Price (1768, p.413,419) observes "improbabilities *as such* do not lessen the capacity of testimony to report truth" and "between *impossibilities* and *improbabilities*, however apt we may be to confound them, there is an infinite difference." Newspapers, for example, are in the business of reporting testimony concerning unusual events and there is general acceptance of the truth of stories in credible newspapers. Directly addressing Hume's attack Price observes that improbabilities "have no *direct* and necessary operation upon [truth of testimony], and should not be considered as a *counter-evidence* invalidating, in proportion to their degree, its reports." In terms of a Bayesian calculation, Price argues that the prior probability for a miracle does not impact the truthfulness of testimony due to the equal prior probabilities *P[M] = P[~M]*.

5. Price, Miracles and Dissenting Theology

Price makes a number of arguments in Dissertation IV concerning miracles. The Bayesian argument is only introduced to undermine the inductive probabilities used to define a miracle in Hume's attack. The primary argument is theological, using 'rational intuition' to determine that the virtuous nature of Christian teaching provides reasonable evidence for the truth of Christian miracles. In opposition to modern interpretations that reference only the Bayesian element, it is theology not Bayesian analysis that is the essence of Dissertation IV. Being a dissenting minister, Price was not compelled to adhere to the doctrinaire Christian miracle that was the immediate target of Hume's attack. As such, Dissertation IV permits Price to be situated within the evolution of Protestant theology going back, at least, to Newton. Earlier contributors to this evolution were significantly impacted by doctrinaire restrictions, such as Samuel Clarke.[15] Instead of recognizing Price (1768) for the relatively small component related to the Bayesian argument, this paper argues that it is more appropriate to view Dissertation IV as the last of four religious dissertations where the

role of 'rational intuition' in the dissenting Protestant theology of Richard Price is detailed.

The enigma posed by the treatment of Richard Price in 18th century intellectual history has theological roots. Price (1768) represented the culmination of a century long evolution of religious dissent that began with the natural philosophy discoveries of Isaac Newton and his tracing the ancient roots of dissenting Christian theology to the Arians, an anti-trinitarian early Christian sect that denied Jesus was co-eternal with God. Immediate followers of Newton – William Whiston (1667-1752) and Samuel Clarke (1675-1729) – also felt Arianism was the doctrine of the early Christian church. Following Harrison (1995) and Pfizenmaier (1997), while the prototypical Newtonian view on the divinity of Jesus was Arian, simple descriptions of this theological position are illusive. The general position differed from that of 'free thinking' religious dissenters, represented in Newton's time by the Socininans, that denied the divinity of Jesus and emphasized Jesus as a prophet and conveyor of God's word. Being a prolific Bible scholar, Newton developed an involved dissenting theology based on detailed study of Scripture and early Church history. Though Newton's important intellectual stature prevented his personal variant of anti-trinitarian dissenting theology from being publicly displayed, the Boyle lectures provided the venue for the development and presentation of Newton's natural philosophy and, indirectly, his theology. Over the course of a series of lectures and writings, considerable evolution in both dissenting Protestant and liberal Anglican theology was inspired by Newton and his followers.[16]

Though relatively close in historical time, the religious and social environment of Newton, Whiston (Boyle lecturer 1707) and Clarke (Boyle lecturer 1704-5) differed significantly from that of Price. The collapse of the divine right of kings followed by the Revolution Settlement with milestones stretching from the Bill of Rights in 1689 to the Act of Settlement in 1701, marked a

period of turmoil for the 'High Church'. The persecution of orthodox Anglican beliefs and practices under the Commonwealth was still fresh in the minds of many. The return of the Anglican party to power in the Cavalier Parliament marked the revival of the High Church position in the English political structure. However, the Revolution Settlement eroded the legal and political powers of the Church and marked the political emergence of a landed Whig oligarchy that was populated by "freethinkers, materialists, atheists and deists". After a generation of struggle, there was a need to renew the Anglican theology of the Church of England. This social, political and religious initiative coincided with the dissemination of the natural philosophy contained in Newton's marvelous contributions. During this period, there emerged an uneasy alliance between the Anglican church and the loose collection of intellectuals that composed the early beginnings of "the Newtonians", e.g., Jacob (1976, 1977); Dobbs and Jacob (1995).

The reign of Queen Anne (1702-1714) was marked by the High Church alarm of "Church in Danger" (Stewart 1981). It was during this reign that Newtonian experimental science emerged into the public conscience. At the same time, adherence to the trinitarian doctrine was still a required litmus test for Anglican orthodoxy, a prerequisite for important political and university positions. The Newtonian struggle with the metaphysics of natural philosophy is evident in the Boyle lectures of Clarke and Whiston. While these lectures were at least superficially consistent with orthodox Anglican teaching, both Clarke and Whiston subsequently published a number of works which were clearly of Arian persuasion, with Clarke, *The Scripture Doctrine of the Trinity* (1712), and Whiston, *Primitive Christianity Revived* (5 vols., 1711-1712), being especially significant efforts. Following Newton, this evolution was based primarily on scriptural interpretation and examination of early Church documents. The inherent metaphysical conflicts between trinitarianism and Newtonian

natural philosophy that were raised in early writings were no longer a central concern. Fears of such unorthodox scriptural interpretations by such prominent Newtonians fueled the persecution of Clarke and Whiston. In 1710, Whiston was removed as Lucasian professor at Cambridge, a position he obtained on Newton's retirement in 1703, and was expelled from the university after a well-publicized hearing. The controversy culminated two years later in Clarke's humiliating promise to the Upper House of Convocation not to preach or write any further on the topic.

By the appearance of the Price (1768) the character of religious orthodoxy in England had softened considerably. While Whiston was denied entry into the Royal Society, ostensibly for publishing scriptural interpretations that denied trinitarian doctrine, Price suffered no such sanction, despite publicly detailing a particular interpretation of Arian theology.[17] Between Newton and Price there was considerable evolution in British Christian theology away from trinitarian Anglican orthodoxy and towards those of religious dissenters.[18] Price (1787) identifies common beliefs held by all Christians, which he identifies with Socininan, Trinitarian and Arian theologies. Included in these common beliefs are: "That Christ was sent of God; that he is the true Messiah; that he worked miracles, and suffered and died and rose again as related in the four Gospels." Given this common ground, on the specific issue of miracles, there was considerable divergence. Clarke's theology, for example, has been classified as Trinitarian, not Arian as in most historical account, e.g., Pfizenmaier (1997a). There was also divergence within each general theological approach , e.g., not all Arians accepted a non-interventionist view of miracles. Finally, some dissenting Christians could not be classified as either Socininian or Arian.

Confronted by Hume's attack, the problem for Price was: How to rationally reconcile this common Christian belief in miracles with Newton's mechanical universe governed by the immutable laws of

nature? Maintaining that miracles were observable and interventionist, orthodox Catholic doctrine was indefensible against such a sceptical attack. Some alternative approach to defining miracles was needed to provide a rational defence. The most apologetic Christian approach to Hume's attack was to avoid defining a miracle as a violation of the laws of nature and Price (1768) follows Clarke in proposing this solution at certain points. However, as the only response to Hume's attack, such a definition would only satisfy those with Socininian inclinations. Those of Arian persuasion, such as Price, would be unwilling to reduce the resurrection of Jesus to being little more than a myth or story. Price needed the more common Protestant claim that Christian miracles only happened in ancient times, at a particular place. This approach was a considerable evolution from the interventionist miracles that were observable violations of nature proposed in the 17th century by dissenting Christians writing prior to Clarke, such as Robert Boyle and Thomas Sprat, who went so far as claim superior ability to judge whether a miracle had happened due to deeper knowledge of the Newtonian natural philosophy (Harrison 1995, p.536).

   In the period between Newton and Price, the use of different approaches to defining miracles was common in Christian writings. In the Boyle lectures, Clarke concludes about miracles: "'Tis not therefore a right Distinction; to define a *Miracle* to be That which is against the *Course of Nature."* Writing long before Newton proposed his version of the laws of nature, St. Augustine and Thomas Aquinas both emphasized the 'unusual' character of miracles and that approach is adopted by Clarke, both in the Boyle lectures and in the dispute with Leibnitz over miracles.[19]   It is apparent from Dissertation IV that Price also recognizes that miracles need not be defined as violations of laws of nature (Price 1768, p.436-7):

          a miracle, according to common opinion, implies a *violation* or *suspension* of the laws of

nature. But, in reality, this is by no means necessarily included in the idea of a miracle. A sensible and *extraordinary effect* produced by a *superior power*, no more implies that a law of nature is *violated*, than any *common effect* produced by *human power*.

Arguments involving different definitions of miracles are interwoven with arguments from prophesy. As a rational basis for Christianity, prophesy is based on scriptural interpretation, especially the book of Revelation (Harrison 1999; Fruchtman 1983): "in any instance, a scripture prophesy is fulfilled, an unprejudiced person must be impressed. It affords, not only a demonstration of the credibility of miracles, but, in some degree, an *actual exhibition* of them" (Price 1768, p.381). Absent the Bayesian element, the overall approach of Price's reply to Hume's attack follows lines similar to previous efforts aimed at countering materialist, empiricist, deist and atheist attacks on miracles dating back at least to Clarke.

6. Miracles and Rational Intuition

  Karl Pearson (1978, p.373) attributes much of Price's doctrine on miracles to Samuel Chandler. Though the source of this claim is not provided and the voracity not supported by other biographical sources, the Stoic characterization of Price's 'doctrine of miracles' is accurate: miracles are to be believed if support is given to a messenger of God, such as a prophet, who is a teacher of just and rational views that lead to the practice of virtue and goodness. In Price's words: "A revelation to instruct and reform a sinful and degenerate world is so far from implying any absurdity, that it is an effect of divine goodness which might reasonably be hoped for" (Price 1768, p.438). In a similar statement, Price (1768, p.439) observes:

> If we reject the miracles mentioned in the New Testament, it will not be possible to give any tolerable account of the establishment of such a religion as the christian among mankind, by a few persons of no education or learning, in opposition to all the prejudices and powers of the world.

Unlike the Bayesian argument, these are not words submerged in a footnote. Rather, these and other similar statements are contained in the body of the text. As such, despite some Bayesian tendencies to the contrary, Dissertation IV was written more as a contribution to moral philosophy and theology than as an application of Bayesian analysis to Hume's attack.

The doctrine of miracles proposed by Price depends on accurate identification of 'virtue and goodness', a problem that is central to moral philosophy. For Price (1768, p.459-60), the primary source of teaching on "divine goodness" is the New Testament:

> It is not conceivable, that any one can read the New Testament, and observe with what a force and purity, before unknown, it teaches morality and natural religion; the sublime and singular character it has drawn, without the least appearance of art and effort, in the history it gives of the life, discourses and Miracles of Jesus Christ; and the spirit of piety, goodness, love and heavenly mindedness which breathes through all its parts

The connection between the divine mind and the human mind being revealed through miraculous works is a key element in the Stoicism Price shared with other dissenting Protestants, e.g., Craig (1987). Stoics maintained the pursuit of knowledge and the exercise of reason was a duty owed to "the Deity". While free thinkers of this period applauded Newton's scientific achievements as evidence for the use of reason alone to decide philosophical issues, this approach was actively opposed by dissenting Christians such as Price. Precisely how this opposition was expressed differed across individual adherents to dissenting theology.

Given the miraculous revelation of the New Testament, Price's rationalist moral philosophy maintained that being patient and virtuous will be rewarded by the just and benevolent divine Spirit (Thomas 1977, p.40). Contrary to empiricists and sceptics such as Hume, moral judgments involve a non-arbitrary intuition about necessary truth. While this moral philosophy created some difficulties with respect to Arian theology,[20] Price was convinced of the need to be open to the rational

consideration of alternative moral and theological arguments: "It would be much better, if christians ... would take care that their own faith is the result of honest enquiry, and at the same time study to demonstrate the excellence of their religion by the excellence of their tempers and lives" (Price 1768, p.463). Continuing the Newtonian tradition, for Price honest enquiry included the active study of natural science which is of great assistance in revealing the divine mind.[21] Price compares the miraculous discoveries in natural science with the "improbability" of a miracle defined as a "sensible and *extraordinary effect* produced by a *superior power*"(Price 1768, p.437):[22]

> there is nothing of the improbability in miracles which some have imagined. I may even venture to say, that they have in them a much less degree of improbability, than there was, antecedently to observations and experiments, in such *phenomena* as *comets,* or such powers as those of *magnetism* and *electricity*.

The reference to comets extends the scientific enterprise well beyond the scope of the contributions of Newton.

Starting with his first book (Price 1758), Price presents an evolution of moral philosophy that has been described as 'rational intuitionism', e.g., Thomas (1977, p.vii). This reference is intriguing because of the connection to modern 'rational intuitionists', a loosely defined group that, almost certainly, includes the 'ethical intuitionists' such as H.A. Prichard and W.D. Ross. Contrary to the utilitarians, ethical intuitionists proposed a different answer to the question: which action is right? While utilitarians would always choose the action that produces the most good, ethical intuitionists identify "a number of distinct and irreducible basic duties or moral principles, all of which can be relevant in determining whether some action is right" (McNaughton 1996). This debate which took place mostly between WWI and WWII, prompted H. Joseph to observe of the ethical intuitionists that "our obligations are not a heap of unrelated obligations" (Joseph 1931, p.92). While the likes

of Prichard and Ross were not able to deal with this type of criticism, Price can address such criticisms theologically with the revelation of virtuous and good actions in the New Testament. There is direct connection between moral philosophy and theology that is not available to modern rational intuitionists.

Since the positivist onslaught initiated by Comte in the early 19th century, theological and metaphysical solutions to problems in moral philosophy and epistemology have been generally disregarded, especially by the scientific community. The predictable reaction is to disregard all philosophical solutions that depend inherently on theological or metaphysical foundations, even though there may be elements of a particular philosophy that do not depend substantively on theology. It is often too frustrating to disentangle the various threads of the argument to focus on those elements with theological underpinnings that have modern relevance outside religious studies. In this process, some debates on topics of modern relevance have been ignored. The debate between Price, the rational intuitionist, and Frances Hutcheson, the utilitarian, on whether benevolence was the whole of virtue, is one such debate. Translated into modern times, this has relevance to the debate between Ross and Pritchard, on the one side, and the utilitarians, represented by G.E. Moore, on the other. This debate was concerned with the question: is there more than one fundamental moral principle? Utilitarians argue for a single rule: choose that action which produces the most good for the greatest number. Rational intuitionists argue that moral judgment is more complex. Intuition is required to balance the claims of different, and irreducible, moral principles (Thomas 1977, p.74-5).

Contrasting these two debates is revealing. The central issue is the same: is the utilitarian formula a valid guide for making moral decisions? The statement of the utilitarian rule by Hutcheson is

much the same in both debates: *'that action* is *best* which produces the *greatest* happiness for the *greatest numbers*', e.g., Raphael (1969) and Hutcheson (1738). In a theological context, this implies that the providence of God depends only on the happiness of his subjects. In an individual context, it implies that virtue flows only from the consequences of actions. Price did not agree with either of these positions (Thomas 1977, p.74-5). Though beneficence may be the most important virtue, it does not necessarily take precedence. Some individual actions are undertaken without considerations of immediate beneficial consequences. For Price, such actions have a religious basis. The duty of prayer, the obligation to worship and be faithful to God, the obligation to keep promises and the duty to "think rightly about disputed points of Christianity" are all actions that are not undertaken for immediate beneficial consequences. In contrast, Ross has a list of duties involving fidelity, reparation, gratitude, justice, beneficence, self-improvement and non-maleficence (McNaughton 1996, p.435-6). While Hutcheson takes an empiricist's approach to the utilitarian rule, Moore recognizes that apprehension of natural properties is insufficient to determine 'good' activities and actions, intuition is also required (Lang 1964, esp. p.295-6).

Unlike the modern rational intuitionists, Price is able to make sense of the 'unconnected heap of duties' by referencing the necessary truths of Christian teaching. In addition, the failings of moral philosophy derived from reason alone have provided a more general recognition of the role of intuition in determining whether an action is good or right. As such, the connection between modern 'rational intuitionism' and the 'rational intuition' of Price is muted. As a rationalist, Price maintains that rational and objective moral judgments can be made independently of sensory perceptions; as an intuitionist Price holds that reason alone cannot provide sufficient guidance for moral judgment. The epistemology associated with the moral order implied by the rational intuitionism of Price

features prominently in the response to Hume's attack. The counterattack on the sceptical empiricism of Hume is evident (Price 1768, p.390):

> it is necessary first to consider the nature and foundation of that assurance which experience gives us of the laws of nature. This assurance is nothing but the conviction we have, that future events will be agreeable to what we have hitherto found to be the course of nature, or the *expectation* airing in us, upon having observed that an event has happened in former experiments, that it will happen again in *future* experiments. This expectation has been represented as one of the greatest mysteries, and the result of an ingenious and elaborate disquisition about it is, that it cannot be founded on any reason, and consists only in an association of ideas derived from habit, or a disposition in our imagination to pass from the idea of one object to the idea of another which we have found to be its usual attendant.

Intuition is required to determine the laws of nature from observation: "An experiment which has often succeeded, we expect to succeed again, because we perceive intuitively, that such a constancy of event must proceed from something in the constitution of natural causes" (Price 1768, p.391). For Price, such arguments extend naturally to assessment of testimony because: "Testimony is truly no more than sense at second-hand" (Price 1768, p.416).

   Even though the concept is not generally accepted in modern academic discussion, situating the human mind within the divine mind is still a powerful method of resolving the confusion arising from the use of intuition to produce both empirical and moral judgments (Craig 1987). For Christians, divine teaching revealed in the New Testament is intuitively true and good; key problems in epistemology and ethics are transformed into problems in theology. In supporting the Stoic and Newtonian goal of establishing a rational foundation for Christian religion, throughout his life Price stressed the importance of thinking rightly about the disputed points of Christianity between the primary theological viewpoints of his time: Trinitarian, Arian and Socinian. Though the essence of a rational Christian response to Hume's attack can be found in earlier sources, such as the Boyle lectures by Clarke, there were significant institutional restrictions on the ability to make such non-

doctrinaire theological arguments clearly and forcefully early in the 18th century. A similar comment applies to Hume's attack. Miracles are an essential motivation for Christian teaching and Price was subject to considerably less restriction in launching a response than historical precursors such as Newton, Clarke and Whiston.

7. Conclusion

Due to the modern significance of Bayesian analysis, Dissertation IV of Price (1768) is still remembered for one of its varied and interesting insights. The bulk of this largely theological contribution is usually ignored; the other arguments against Hume's attack go unrecognized. An integral part of a much larger philosophical project, Hume's attack is about the use of inductive empiricism to infer causes from effects, a problem that inspired Bayes (1763) and still generates intellectual debate. This paper situates the theological arguments made in Price (1768) in the evolution of dissenting Christian theology after the discoveries in natural philosophy by Isaac Newton. By carefully addressing the character of miracles, Price was able to avert the devastating sceptical implications of Hume's attack for doctrinaire Christian miracles, that were required to be both contemporaneously observable and interventionist. In the process, Price was able to sustain a theological basis for the use of 'rational intuition' to resolve moral questions, a problem that has plagued modern moral philosophers.

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NOTES

1. The Reverend Thomas Bayes (1702?-1761) earned the eponym 'Bayes's theorem' for results appearing posthumously in Bayes (1763). The reproduction available in Deming (1940) contains Molina (1940). Stigler (1983) argues unconvincingly for an earlier beginning for Bayes's theorem. The history of Bayes's theorem is thoughtfully examined in Dale (1991).

2. This concurs with Owen (1987, esp. p.196): "My main purpose has been to show that Hume's argument has a larger significance than is generally realized, that Price saw this and has been unjustly neglected, and that their debate is extremely similar to an important modern issue." Similarly, Earman (2000, p.24): "The name of Richard Price is largely unknown to modern readers."

3. Sources that focus on the political element of Price's contributions include: R. Thomas (1924); Laboucheix (1970); and, Fruchtman (1983). A listing of a number of sources discussing Price's contributions to political theory is available in Thomas (1991) and Molivas (1997).

4. "Dissertation IV. On the Importance of Christianity, the Nature of Historical Evidence and Miracles" in Price (1768). Changes appeared in "Of Miracles" in subsequent editions to Hume (1748). Earman (2000, p.77) observes: "Hume commentators have generally neglected the clues to Hume's intentions offered by these changes."

5. Recent contributions include: A. Tucker (2005); Earman (2000); and Holder (1998).

6. Essential works of David Hume (1711-1776) are Hume (1739-1740) and the more influential Hume (1748). Buckle (2007) provides a brief but excellent overview of Hume. In the realm of moral philosophy and ethics, sources on Price include: Raphael (1947); and, W. Hudson (1970).

7. At the end of his life, Price also came out in support of the French Revolution, an event that eventually evolved into a blood-bath. However, Price died in 1791 before the French Revolution could take its darkest turns. In the end: "Price was a mild and gentle man whose strength of character led him into a position of great authority, yet without any trace of self-seeking. In some ways he was remarkably far-sighted and in other remarkably wrong-headed, but with it all honest and candid" (Ogborn 1962, p.97).

8. All quotes being provided for Price (1771) are from the third edition (1772). In turn, the specific quotes being directly quoted by Price are from a 22 July, 1772 article in the *Public Advertiser*. Price refers to "the ingenious proposer of this scheme" set out in the *Public Advertiser*. However, Pearson (1978, p.403) observes that the "ingenious proposer" was actually Price himself, the author of the article in the *Public Advertiser*.

9. The works of Pierre Simon, Marquis de Laplace (1749-1827) on inverse probability start with two minor papers in 1774 and conclude with *Theorie analytique des probabilities* (1820, 3rd ed.). Dale (1991, ch.6) provides a detailed examination of these contributions.

10. Writing after Laplace, in *Laws of Thought* (1854) Boole refers to this case as "equal distribution of ignorance" and is able to demonstrate inherent difficulties with this assumption for solving general inverse probability problems.

11. This follows Harrison (1999, esp. p.241-2). The statement of Hume's attack can be made stronger by observing that the evidence for second hand testimony of Christian miracles was written by individuals other than the actual witnesses to the miracles. This complication to Hume's attack was not addressed by Price or Hume, though Bible scholars such as Newton or Price would have known this.

12. Dale (1991, p.113) traces this discrete version of Bayes's theorem to M. Condorcet, *Memoire sur le calcul des probabilities* (1781), pt. 5.

13. Zabell (1988; 1989) and Dale (1991) are exceptions. Some sources make references to Price's contribution without much detail, e.g., Owen (1987, p.195): "[Price's] point entirely undermines Hume's use of prior probabilities in his discussion of rationality of belief in miracles based on testimony." For relevant sources for the contributions of Laplace and Condorcet on this issue see Sobel (1987, esp. n.2); and, Dale (1991, esp. n.8 and chaps. 5-6).

14. The closest Price comes to directly identifying the condition $P[M] = P[\sim M]$ is the following: "the improbability of event here mentioned, must mean the improbability which we should have seen there was of its happening independently of any evidence for it, or, previously to the evidence of testimony informing us that is *has* happened"; Price (1768, p.405).

15. Various sources identify Voltaire's arrival in London in 1726 as a symbolic demarcation in the types of intellectual and political problems faced by Enlightenment thinkers in England, e.g., Jacob (1977, p.4).

16. See: Dobbs and Jacob (1995); Jacob (1976); and, Jacob (1977, p. 1-25). These lectures were created in 1691 by an endowment in the will of Robert Boyle and were given continuously from 1692-1714, with occasional breaks until 1732. The lectures were given sporadically during the 19th century and were revived in 2004.

17. The most accessible source for these views is Price (1787), a collection of sermons, many of which were delivered well prior to 1787.

18. Following Pfizenmaier (1997), there is some disagreement over whether Newton was an Arian and even some support for the position that, in later life, Newton came to support trinitarian doctrine. Whatever the case, there is no disagreement that Newton was a dissenting theologian with an individually determined religious perspective.

19. On Clarke versus Leibnitz see Vailati (1995). On the Newtonian definition of miracles, see Harrison (1995).

20. On the conflict between rationalism and Arian theology see Thomas (1977, p.38-40).

21. In relation to prayer, Price discusses this point in Dissertation III, Price (1768, p.255).

22. The process of the Deity producing these effects is detailed in Dissertation I.

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Nevertheless, the connection between Bayesian Statistics and Decision Theory is developed. Moreover, the author emphasizes the increasing importance of computational techniques." (Krzysztof Piasecki, Zentralblatt MATH, Vol. 980, 2002). From the Back Cover. Winner of the 2004 DeGroot Prize. Robert develops both the decision theoretic background of Bayesian statistics up to the level of The Theory of Point Estimation by Lehmann and MCMC computation including practical implementation issues. The author is to be commended for writing a book that contains very advanced material from mathematical statistics, but the book can be used by a wide audience since the parts on Bayesian computation are easily accessible. Parmigiani, G. (Giovanni) Decision theory : principles and approaches / Giovanni Parmigiani, Lurdes Inoue. p. cm. Includes bibliographical references and index. ISBN 978-0-471-49657-1 (cloth) 1. Statistical decision. 3 Utility 3.1 St. Petersburg paradox 3.2 Expected utility theory and the theory of means 3.2.1 Utility and means 3.2.2 Associative means 3.2.3 Functional means 3.3 The expected utility principle 3.4 The von Neumann–Morgenstern representation theorem 3.4.1 Axioms 3.4.2 Representation of preferences via expected utility 3.5 Allais'. 10.4.1 The well-calibrated forecaster 10.4.2 Are Bayesians well calibrated? 10.5 Exercises.