Genesis of Selected Mathematical Notations

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ABSTRACT

This study examined the meaning and purpose of notation, historical development of the operational symbols such as addition, multiplication, subtraction and division. Also, the Relational Symbols of Variables (unknown Quantities) and their powers, Euler and Leibnitz as great notation builders were discussed. It was concluded that, the creation of notations and their uses in mathematics has made mathematics to be not only international Language of Science but also has been employed to solve human, scientific, technological, social and political problems.

Keywords: Mathematical notation, operational symbols, Science

INTRODUCTION

Most students grow up believing that, mathematics textbooks were handed over fully worked out and they thought that, the only struggle involved is for them to master it. Shirley (1986) argues that mathematics teachers should correct this wrong impression of most of the students by occasionally informing them how mankind struggled for several thousand years to develop the present Hindu-Arabic numeral which we now take for granted. How the operational symbols +, -, x and ÷ evolved to its present form through Egyptian, Greek and Arabic civilizations. Shirley (1985) and Ezekute and Ihezue (2006) argues that, mathematics teachers should show the students that mathematics is for the real world and its applications cut across all human endeavours as corroborated by Abe and Egbon (2011) that mathematics is used to solve real world problems because; mathematical language is more efficient and less bulky than the written words. It is more difficult to cheat conclusion with mathematical argument. The result of a mathematical debate are precise and depend only on the initial assumptions, for a given set of assumption the mathematical conclusions are accurately expressed and their result cannot be argued. With a mathematical description, it is possible to arrive at optional solutions which would not be obvious without analysis (Taha, 2006).

Also, Abe and Egbon (2011) and Shirley (1986) posit that mathematics should not be perceived by the student as a bunch of useless and boring nonsensical subject. To avert this, during the introduction of new topics, telling how this topic developed historically or mentioning biographical details of a mathematician who worked on the topic and even mention problems that a particular topic helped to solve, to serve as a motivation or a justification step is very helpful. At the end of a lesson, a teacher should offer another place for introduction of some mathematical symbol or notations. The main object of this study therefore is to assess the genesis of selected mathematical notations.

MATHEMATICAL NOTATIONS AND ITS PURPOSES

Notations are symbols depicting relations; operations quantities etc. Bram (1971) posted that symbols denote various signs and abbreviation used in mathematics to indicate entities, relations or operations. Lapedes (1978) defines notation as the use of symbols to denote quantities or operations. This implies that notation is an abstract mathematical symbol used to represent quantities as in numerals, relations or operations. In this study however, the mathematical notations involving operations or relations are restricted strictly to those used in senior secondary curriculum in both General and Further mathematics. Such operational notations are +, - x ÷ and that of relational notions include = <, >, F(x)

Notations were introduced because they help in expressing ideas in a more concise form. It serves as a simple shorthand and as an aid to thought, Boyer (1968); Ball, Catt and Copper (1978) have argued that development of notations gave the development of mathematics a leap forward although illiterate people often do some simple reckoning. Mathematic advanced rapidly only when written symbols were employed. This clue is in consonance with what Adewuye (1992) posits that as soon as men were able to symbolize, they created a big gap that justified man's conceptual capacity and the apparent absence of it in other animals. The development of notations (symbols) involved three different overlapping stages. Stage one is described as rhetorical stage in which problems and their solutions were stated in words. An illustrative example of this state can be found in the Rhind Papyrus (c.a 1700BC). The second stage was called syncopated stage in which simplification was obtained by using abbreviation for words. The third stage are called the symbolic stage in which the word or their abbreviations are replaced by symbols (Boyer, 1968; Bell, et al 1978; Adewoye, 1992 and Ezekute and Ihezue, 2006).

HISTORICAL DEVELOPMENT OF THE OPERATIONAL SYMBOLS

Addition And Subtraction: The symbol "add" is part of legs walking the right to left (>>) while that of "subtract" is a part of legs walking the opposite way (<<). The Greek mathematician indicated addition of term in the appropriate juxtaposition of the symbols for the terms and subtraction was represented by a single letter abbreviation placed before the terms to be subtracted. This shows that, in Greek period, there was no special symbol for the operations of addition and subtraction. However, during this period, all operational and relational symbols were not in existence (Eves 1964). Nicolas Chuquet posits that the four fundamental operations were indicated by the words and phrases; plus, mois, multipliers par and partyr par, sometimes addition and subtraction were abbreviated as a *P* and *m*. subsequently, Descartes, Rene 1596-1650 avoided the use of negative quantities, which he called "false" though he did not forbid their usage. The first appearance of addition sign "+" and subtraction sign (usually denoted by an hyphen "-") was in Arithmetical book published in 1489 by Johanna Wideman (ca 1460) in Bohemia. The sign "+" was then called "signum auditorium to denote excess while the sign "-" was signum subtraction to denote deficiency. He further demonstrated that, the plus sign is derived from constraction of a handwritten abbreviation of the latin word *et* meaning add while the subtraction sign i.e. minus sign is possibly, contracted from the Latin abbreviation *im* for minus. Stifle (ca 1487-1567) popularized the German symbols + and - at the expense of the Italian symbols *p* and *m* notation (Dedron and Itard, 1973; Bell, *et al*, Bram, 1971and Eves, 1964).

Multiplication and Division Notations or Symbols: Viete Francois (1540-1603) used "in" for multiplication and fraction line for division while Leibnritz (1664-1786) adopted and employed the sign "n" for multiplication and "u" for divisions while William Oughtre (1574-1600) an English mathematician introduced the symbol "x" for multiplication. The modern symbol for division first appeared in 1659 in Algebra by Johann Herinrich Rahn (1622-1676), a Swiss. Sherman (1972) argues that the recognized symbol for division was for four hundred years (400 years) used to depict subtraction by German, Dutch and Swiss. But it was the swiss mathematician J. H. Rahn who found two signs - and ÷ for subtraction and divisions.

The Relational symbols: <, = and >, Thomas Harriot (1560-1621) was the first to use the sign "<" for less than, ">" for greater than, but these symbols wer not immediately accepted by mathematics writers. Rene Descartes (1596-1650) introduced the use of the symbols "x" for equality in his book La Geometric, while Robert Recorde (1510-1588) popularized the use of the present equality sign "=" in his book, the Whetstone of witte. Viete, Francois (1540-1603) used the abbreviation "ae" from equalis for equality (Easton, 1966). However, it was only a hundred years later that the present sign of equality as it was, triumphed over rival notations due to the adoption of the sign by Newton and Leibnitz.

Variable (unknown Quantities) and their powers: In Diopharitus book, the Arithmetic, this is a systematic use of abbreviations for powers of numbers and for relationships and operations. An unknown number is represented by a symbol resembling the Greek letter "*s*", the square of this appear as Δ^y the cube as k^y, the fourth power called the square-square as and the fifth power or square cube as Δk^y and the sixth power or cube-cube as $k^y k$. Diophanuts wrote $2x^4 + 3x^3 + 4x^2 + 5x -6$, a polynomial of degree 4 as *SS* (3 x 5Ns 4U6) where *S* denote the "square", *N* is the "cube", x the "unknown", *m* the "minus" and *U*, the "Unit". The main difference between the Diophantine syncopation and the modern algebraic notation is the lack of special symbols for operations and relations as of the exponential notation. Nicole Oreseme (1322 -1383) posits the use of special notations for fractional powers in his Algorisumus proportium. There are expressions such as:

P	1
1	2

Which depicts "one and one half proportion, that is, the cube of the principal square root and forms such as:

$$\frac{1 - P - 1}{4 \cdot 2 \cdot 2}$$

Viete Francois (1550 -1603) used a cube for A^3 and A^2 as a quadralus. But William Oughtred (1574 -1660) used Aqqc for A^7 meaning A squared cubed. Burgi (1552 -1630) showed powers of unknown by placing Roman numerals over the coefficients. That is $X^4 + 3x^2 - 7x$, would be written as:

$$\overset{iv}{1} + \overset{iii}{3} - 7^{i}$$

And Stevin (1598 -1620) wrote his own by:

$$\overset{(4)}{1} + \overset{(2)}{3} - \overset{(1)}{7}$$

He further postulated that, ½ in a circle would mean square root and $\frac{3}{2}$ in a circle would depict square root of the cube, while Girard John (1590 -1633) adopted the circle numerical notation for powers. Christopher Rudoff in 1525 introduces radical sign $\sqrt{\ }$ for square root, it also stands for radix (root) in German word and *VVV* stands for cube root, *VVVV* for fourth root, while Michad stife (1486 -1567) used 1A, 1AA 1AAA for one unknown, square of an unknown and its cube (Grant, 1978).

Euler and Leibnitz as great genesis of Notation Builders: Euler - leonard (1707 - 1783). Boyer (1968 and 1969).posited that Euler was the greatest notation builders of all times and he was well known for

(i) Introducing the letter "e" to depict the base of a system of natural Logarithms for example Log eq

(ii) Exposing the use of notation $\prod$ as the ratio of the circumference of the circle to the diameter.

(iii) Initiating the use of "I" to indicate -1 and formulated the formula $e + 1 = 0$ which could be found in his book entitled: Introduction and Analysis Infinitorum (in 1748).

(iv) Initiating the use of *a, b, c* for the sides of a triangle and *A, B, C* for the angle of a triangle facing the sides *a, b, c* respectively.

(v) Euler discovered the formula 4rRs = abc, where r is the radius of inscribed circle, *R* the radius of the circumcircle and $S = \frac{(a + b\ c)}{2}$ the semi-perimeter of triangle ABC.

(vi) Initiating " $\sum$ " sigma notation for summation and notation f(X) indicating a function of X (Struik, 1967).

According to Boyer (1959 and 1968), Leibnitz, Gottfried Wilhelm (1664 - 1786) was associated with the introduction of the following:

(i) Dot for multiplication and for proportion

(ii) The equality sign was popularized by Leibnitz along with Sir Isaac Newton. This equality sign was introduced by Recorde while Descartes introduced " ∞ " as equality sign.

(iii) "*F*" is similar to and "=" "for is congruent to", are all due to Leibnitz

(iv) *dx* and *dy* for smallest possible differences (differentials) in *x* and *y* was introduced by Leibnitz. Initially, he had used and he simply wrote *Omn.y* (or all

y's) for sum of the ordinate under a curve, but he later used the symbol *fy* and later *fydy*, the integral sign being an enlarged letter for sum. He further argued that finding tangent required the use of "calculus differentials". Finding areas under a given curve or quadratures required "calculus integrals" and from these phrases we got the words "differential calculus" and integral calculus.

CONCLUSION

The creation of notations and their uses in mathematic has made mathematics to be not an international language of science but also has been employed to solve human, scientific, technological, social and political problems. Isaac Barrow (1630 -1677) owning to the importance of mathematics, he now described mathematics as "the foundation of science and the plentiful fountain of merits to human affairs". Napoleon argued that, "the advancement and perfection of mathematics are intimately connected with the prosperity of the state". Based on these merits associated with mathematics, students should be made to think mathematically by developing a creative questioning skill in them which can be attained by putting across some historical ideal like how mathematical notations/symbols developed and introduced by man in the ages past.

REFERENCES

Abe, T .O and Egbon, F . O. (2011). Mathematics Education as a panacea for Global Societal Challenges in Nigeria. Journal of Research in Education and Society. 2(2), 74 -79

Adewoye, T. O. (1992) Cultural basis for mathematical development, ABACUS, 21 (1 _.106-119).

Bell, K; Catt, R and Copper (1978). The mathematician' symbols. The human world in library of modern knowledge. Z. London, the Reader's Digest Associatio.

Boyer, C.B. (1959) the History of Calculus. New, Dover.

Boyer, C.B. (1968). A History of mathematics, New York. Wiley.

Bram, L.L. (1971). Funk and Wagnall New Encyclopedia. Fund and Wagnall Inc.

Dedron, P. and Itard H. (1973). Mathematics and mathematicians, Z: methods and problems. London ; transworld publishers.

Easton, Joy, B (1966). A tudor Eulid, Scripta mathematics 27, 339 -355.

Eves, Howward (1964). An introduction to the history of mathematics (4th ed) New York; Hoit, Rinehart and Winston.

Ezekute, O.G. and Ihezue, C (2006). The history of mathematics for Tertiary Institutions (2nd) Onitsha, Symac, printing and Design Ltd.

Grant, A (1978). School methods with younger Children. Ibadan: Evans (Nigeria).

James and James (1976). Mathematics Dictionary (4th ed) New York: Litton Educational

Lapedes D (1978). Mc Graw -Hili dictionary (4th - ed _New York: Litton Educational.

Shirley, L.H (1985). Real mathematics for real world. Paper presented at War against poor achievement in mathematics conference. A.B.U., Zaria.

March 9.shirley L.H.,(1986) History of mathematics in Nigerian mathematics classrooms: Values and problems. ABACUS, 17 (1) 123-133.

Struik D.J. (1967). Common elements in new mathematics programme Lorigins and Evolutions Research. An Introduction 8th edition prentice Hall of India, private Limited New DeIhi.

While a comprehensive list of notation is included in the appendix, that is meant mostly as a reference tool to refresh the reader of what notation means. This section is to introduce the notation to the reader and explain its usage. This is not meant to be a comprehensive or rigorous definition of set theory. We will define a minimal amount of set-theoretical objects, so that the concept of mathematical thinking can be understood. In this book, we will use capital letters for sets and lowercase