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THE IMPLEMENTATION OF VALUE AT RISK (VaR) IN ISRAEL'S BANKING SYSTEM

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The three most commonly used methods for calculating the VaR (Value at Risk) of a typical trading book in Israel's major banks were examined in this study. An analysis of risk factors in Israel shows that the daily changes are not distributed normally, *inter alia* because of Israel's special characteristics: high inflation, indexation, a thin market, and an exchange rate in a band with a positive slope. A comparison of the three most common methods for calculating VaR showed that the capital requirements obtained from the calculations based on the historical and the Monte Carlo simulations, which are not parametric, were smaller than those derived from the calculation based on the variance-covariance method. When accounting for drift (which was found to be positive in the sample period) in the calculation of the latter, the results for all three methods are very similar, however. We also examined three ways of calculating the daily changes in the risk factors: multiplicative (the rate of change), which is appropriate mainly for share indices and exchange rates, additive (absolute change), which is more suitable for interest rates, and a mixed approach, combining the first two in accordance with the risk factor. Our examination indicated that the multiplicative method is far more volatile than the additive and mixed methods, irrespective of the way VaR is calculated. Our main conclusion is that choosing the appropriate way of calculating daily changes is at least as important as choosing the method of calculating VaR.

In recent years there has been growing awareness on the part of bank directors and supervisory authorities of the need to assess market risks, and even to determine a capital requirement against them, in addition to the capital requirement against credit risk.

In April 1993 the Basle Committee recommended a framework for measuring market risks, and in January 1996 a general agreement was drawn up for assessing them and determining capital requirements against them. This agreement, which was adopted by all the countries represented on the committee, was supposed to be put into effect by the end of 1997. According to the committee, banks can assess market risk either by means of a standard model or by using an internal model based on VaR and in which they have a certain degree of freedom in setting various parameters, subject to the approval of the supervisory authorities.

Value at Risk (VaR) is an estimate of total exposure to the various market risks—interest rate, inflation, exchange rates, share prices, etc. It expresses the maximum loss

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expected for a specific position in assets and liabilities that are sensitive to changes in market factors for a pre-set horizon and confidence interval.

In the present study we apply the three methods most commonly used for calculating VaR, as well as the standard method recommended by the Basle Committee, to Israel's banking system. We also examine the extent to which the various models are appropriate for Israel's special characteristics, especially the asymmetry of the changes in these market factors. This derives *inter alia* from Israel's high inflation rate and its indexation mechanisms, which are stronger than in other western countries, as well as from the fact that the exchange rate is managed within a band with a positive slope.

The next section gives a review of the literature concerning the calculation of VaR, a field which has developed in the last few years. In Section 2 we describe the three most commonly used methods for calculating VaR: a historical simulation, a variance-covariance matrix, and the Monte Carlo simulation. Section 3 describes three ways of calculating the daily changes in the price of risk factors. In Section 4 we calculate VaR for the typical trading book of an Israeli bank and compare various internal models with one another as well as with the standard method for calculating VaR. We examine the effect on VaR of the ways of calculating the changes in the risk factors, and undertake backtesting of the various methods of calculation; we also provide some examples of different portfolios (banking books) and their impact on risk. Our conclusions are presented in Section 5.

1. REVIEW OF THE LITERATURE

The literature on VaR has developed greatly in the last few years. Several articles calculate the estimate of market risk on the basis of VaR, using various methods. Lopez (1996) compares these methods, and assesses the accuracy of each one. Jackson et al. (1997) examine the possibility of predicting the variance of risk factors (interest rates, share prices, and exchange rates), and compare parametric and non-parametric methods of measuring VaR, while Crnkovic and Drachman (1996) examine the possibility of predicting all the parameters of distribution of risk factors. Bassi et al. (1996) discuss the difficulties of estimating the probability that exceptional events will occur, and recommend preferring the use of extreme values rather than the accepted methods of calculating VaR. Kupiec (1995) ran backtesting for the VaR model, examining the length of time that passed until the first failure of the estimate, as well as performance tests for the failure rate. At a later stage the Basle Committee adopted these tests as the basis for examining banks' internal models. These studies do not provide clear statistical tests of the accuracy of each of the estimation methods, and most of them deal neither with the capital requirement, which is derived from the various methods of calculating variance, nor with the subject of the use of historical market data rather than simulations.

Several articles[1] compare the standard approach to estimating market risk, as presented by the Basle Committee, with the VaR model, which is based on three main approaches: 1. a historical simulation; 2. the variance-covariance matrix; 3. Monte Carlo simulations.

[1] Hendricks (1996), Pritsker (1997), Linsmeier and Pearson (1996), Jackson et al. (1997), and Aussenegg and Pichler (1997).

These studies examine the advantages and disadvantages of each method as regards the following aspects: time of calculation vis-à-vis accuracy of estimates (Pritsker, 1997); adaptation to different geographical regions (Powell and Balzarotti, 1996); the effect of financial instruments included in the trading book (Aussenegg and Pichler, 1997).

Among the studies that examined the application of VaR models in different countries, that of Powell and Balzarotti (1996), which compares the application of VaR models and the standard approach in several Latin American countries, is particularly interesting. To some extent, the current study takes a similar approach, as there are greater similarities between the macroeconomic characteristics of Israel and those countries than between those of Israel and other western countries (the G10) that use VaR models. Thus, for example, both Israel and the Latin American countries have a high and volatile rate of inflation, a narrow capital market, and considerable government intervention. Powell and Balzarotti (1996) reach the conclusion that the standard approach is preferable to the various VaR models for the purposes of supervisory bodies, but this is merely an assessment.

The various internal models make several assumptions and use different measurement tools, which could produce varying results for the same set of market parameters and positions in the trading book. Furthermore, VaR models are highly sensitive to assumptions and the way data are estimated, especially in the context of assets that are not basically linear, such as derivatives (Marshall and Seigel, 1997). Marshall and Seigel note that this sensitivity, and the freedom banks have to determine the formation of the internal models, could expose the supervisory and banking systems to 'implementation risk.' Thus, the wide variety of points that have to be taken into account when estimating VaR, and the considerable freedom of choice the authorities give the banks that adopt internal models, could give rise to implementation risk, i.e., significant differences in results for identical portfolios and the same market parameters, as is in fact reported by Marshall and Seigel (1997).

2. METHODS OF CALCULATING VaR

In all the methods VaR is calculated on the same schematic basis: first, basic parameters, such as the period of the sample, the holding period, and the confidence level, are set; then the relevant risk factors are selected and risk mapping is undertaken; finally, VaR is calculated. One of the main problems in estimating market risks by means of VaR (and which is not discussed here) is calculating the contribution of each risk factor to the total risk in the portfolio. For this purpose, the effect on the value of the portfolio of a change in each of the risk factors, or in the position of each asset, must be calculated holding everything else constant.

The methods for calculating VaR are usually divided into two: parametric and non-parametric. The former include the use of a variance-covariance matrix and analytical methods,[2] while the latter include historical simulations and Monte Carlo simulations. The three main methods of calculating VaR are reviewed below.

[2] The internal VaR model used in this study is based on analytical methods. Note that these are used to estimate the risk implicit in derivatives, e.g., options or mortgages with an early-repayment option, by means of appropriate pricing models.

Historical simulation

This is the simplest of the non-parametric methods as it is not based on assumptions about the distribution of changes in risk factors, the structure of the markets, and the interrelation between them. At the first stage of this system the daily changes in the various risk factors are calculated for the holding period. Then the effect of the change vectors on the current value of the trading book, i.e., the profit or loss that would result if past changes were to occur today, is examined for each vector.

The value of the portfolio is defined as a function of the risk factors as follows: $F(p_t)$, where F is pricing, p is the vector of the parameters (prices) of each of the relevant risk factors, and t is the index for today; thus, for example, p_j is the price vector for day j, and p_{j+1} is that for the following day. The simulation is made using the moving window method, in which the size of the sample (size of the window) is set, but the period of the sample (starting day and ending day) changes for each day for which VaR is obtained by the simulation.

In accordance with the directives of the Basle Committee, the changes in the risk factors must first be calculated for ten business days during the period of the sample (not less than one year). Then the effect of those changes on the existing portfolio is calculated, in terms of profit and loss, on the assumption that they will occur today. The VaR is the percentile of the distribution of profits and losses arranged in order from the greatest loss for the bank to its greatest profit (best case scenario). This means that according to past data the probability that a loss greater than VaR will be incurred is less than one percent. Another approach suggested by the committee is to calculate daily changes in portfolio value and multiply them by $\sqrt{10}$, in order to get an estimate of the changes obtained during ten business days. The implicit assumption[3] here is that it is possible to use the equality $\sigma_{10} = \sqrt{10} \cdot \sigma_1$, where σ_{10} is the standard deviation calculated on the basis of changes during ten business days, from day t to day $t - 10$, while σ_1 is the standard deviation calculated on the basis of daily changes, from day t to day $t - 1$. Later in this study we examine whether this assumption holds for Israel.

The advantages of the historical simulation derive from the relative simplicity of applying it as well as from the absence of assumptions about the distributions of the various risk factors and the correlations between them. In addition, it is easy to use this simulation for non-derivative financial instruments, such as futures and options. Its main disadvantage is the lack of data, due to the trade-off between a wide and a narrow sample. On the one hand, the wider the sample (a larger window), the greater the probability that it will incorporate data about more extreme events (changes). On the other, a wider sample also includes data from the distant past that are irrelevant to the present situation. Another disadvantage is the lack of stability in the results of the simulation, reflected in daily fluctuations in VaR. If we assume, for example, that the sample used for the simulation is based on three years back from the day of calculation, and that the greatest loss in the portfolio is incurred (through the simulation) as a result of changes that occurred exactly three years ago, the VaR on the day of calculation will be significantly different from that on the next day, as the latter is based on a sample that does not include the day of the large loss

[3] Assuming that changes in risk are independent of one another and have the same distribution.

three years earlier.

When the portfolio is managed on the basis of technical strategies based on past data using a historical simulation to calculate VaR is inefficient, as the results obtained will have a downward bias. This is because when a portfolio-management strategy is based on past data (rather than on extra-sample data) financial instruments are bought and sold in such a way that they would have yielded the most profit if that portfolio had been held in the past.

Variance-covariance matrix

This parametric method is typically based on the assumption that the distribution of the changes in risk factors (yields) is normal. First, on the basis of the historical data, the risk factors are mapped and statistics that characterize the distribution of changes in those risk factors, such as average, variance, and correlations, are calculated. Then, the expected profit and loss on the trading book is calculated for a given probability (at a confidence level of 99 or 95 percent). The assumption is that the distribution of changes in the value of the trading book is normal, because the value of the trading book is obtained as a linear function of its components. The assumption of a normal distribution is a strong one, but is valid only for changes over short periods (one day). In addition, we use only the first derivative of the changes (the delta approach), on the assumption that the convexity and higher moments are negligible if changes occur in a relatively brief time.

Assuming a normal distribution, it is easy to calculate the cut-off value of any percentile of the distribution of profits and losses in the trading book. Thus, $\mu - 2.33\sigma$ is the value that cuts off 1 percent of the distribution (one percentile), where μ is the mean and σ is the standard deviation. This value is in effect the VaR, as one percentile of the distribution of expected profits and losses reflect the worst case as far as the bank is concerned (the Basle Committee chose 1 percent).

The question that arises at this stage is how to estimate the mean and variance of the changes in the value of the trading book. We assume that there are n risk factors, expressed by the vector $X = (x_1, x_2,..., x_n)$, where X^0 reflects the current value of the risk factors and X^1 is their value tomorrow. We also assume that the daily changes have a normal distribution. We mark the average of changes in the risk factors by means of the vector $\mu_x = (\mu_1, \mu_2,..., \mu_n)$, the matrix of correlations between the changes in the risk factors by S_x, and the value of the trading book set as a function of the risk factors by $P(X)$.

The change in the value of the trading book during the measurement period can be presented by means of a Taylor series, as follows:

$$(1) \qquad P(X^1) - P(X^0) = P'(X^0)\cdot(X^1 - X^0) + \frac{1}{2}P''(X^0)\cdot(X^1 - X^0)^2 + ...$$

Using only the first-order condition (delta approximation), and assuming that the other terms are negligible, we can express the changes in the value of the trading book as a linear combination of the random variables (changes in the risk factors) with the weights $\partial P(X_0)/\partial x_i$. As is known, the sum of normally-distributed variables also has a normal distribution.

In our case, the average of the changes in each risk factor tends towards zero as the changes are on a daily basis, so that the distribution of the daily changes in the value of the portfolio is normal, with the following means and variances:

(2) $$E\left[P\left(X^1\right)-P\left(X^0\right)\right]=P'\left(X^0\right)\cdot\mu^T{}_x$$
$$var\left[P\left(X^1\right)-P\left(X^0\right)\right]=P'\left(X^0\right)\cdot S_x\cdot P'^T\left(X^0\right),$$

where μ_x and S_x are, as stated, the vector of means and the variance-covariance matrix of the rates of change in the risk factors respectively. Superscript T marks the transposed vector.

We can now record one percentile of the distribution of profits and losses in the trading book, reflecting the maximum loss with 99 percent probability, as follows:

(3) $$\text{VaR}_{1\%}\left(P,X^0\right)=P'\left(X^0\right)\mu_X^T-2.33\left[P'\left(X^0\right)S_X P'^T\left(X^0\right)\right]^{0.5}.$$

A decided advantage of the variance-covariance matrix over other methods is its flexibility and simplicity of application, and its widespread use. A notable example of this is J.P. Morgan's variance-covariance matrix (known as RiskMetrics™), which is used by various agencies, and constitutes a source of information for many companies. It can also be used to gauge a trading book's sensitivity to each risk factor, and to assess the effect of alternative scenarios with relative ease, by altering the value of z. The main disadvantage of this method is its reliance on three basic assumptions: first, that the changes in each risk factor are distributed normally; second, that the average of the changes in each risk factor are equal (zero) because of the short measurement period (one day); and third, that the connection between changes in the risk factors and those in the value of the trading book is approximately linear.

Hence, the use of this method for a trading book containing financial instruments whose behavior is not linear, such as options, yields biased results. Furthermore, using this method for financial instruments whose behavior is linear may also produce biased results in certain situations. For example, if the exchange rate is managed within a band with a positive slope, and the actual exchange rate approaches one of its limits, the changes in the exchange rate could be in one direction only, in which case their distribution would not be normal.

Another problem discussed in the present study is that of calculating the variance of the changes in risk factors, assuming that this is not stationary. Note that in parametric methods of calculating VaR various assumptions can be made regarding their development in the market, each such assumption requiring a different econometric approach. For example, equal weight can be ascribed to all the observations in the sample (equally weighted moving average), or greater weight can be given to more recent observations (exponentially weighted moving average) by means of a decay factor.[4]

In the first method, the estimate of variance is calculated by: $\sigma^2=\frac{1}{N-1}\sum_{t=1}^{N}(r_t-r^*)^2$, where N is the number of observations in the sample (henceforth, window size), r_t is the rate of change at time t (generally calculated as the log of the change in price), and r^* is the average rate of change. In many countries, and for most parametric series, $r^* = 0$ can be

[4] See, for example, Boudoukh et al. (1997), Ahlstedt (1997), Alexander and Leigh (1997).

taken as a good approximation, so that the expression obtained is even simpler: $\sigma^2 = \frac{1}{N-1}\sum_{t=1}^{N} r_t^2$. The major disadvantage of this approach is that a price shock will have a uniform effect on the variance calculated even after a long time (until a number of days equal to the window size have passed), and then the variance will suddenly drop. This was observed by Alexander and Leigh (1997), who show that the annual variance of the FTSE, which is calculated using this approach (and window size is one year), jumped to 26 percent the day after Black Monday of 1987, remained at that level for exactly a year, and then fell to 13 percent even though there was no significant economic event on that day. In the second approach greater weight is ascribed to more recent events in calculating the estimate of variance; this is done by using the decay factor $0 < \lambda < 1$, as follows: $\sigma^2 = (1-\lambda)\sum_{t=1}^{N} \lambda^{t-1}\left(r_t - r^*\right)^2$.

Here, too, if we assume that $r^* \cong 0$, it can be seen that the nearest observations receive greater weight than the furthest ones: $\frac{r_t + \lambda r_{t-1} + \lambda^2 r_{t-2} + ... + \lambda^N r_{t-N}}{1 + \lambda + \lambda^2 + ... + \lambda^N}$, unless $\lambda = 1$, in which case the first approach is accepted and all the observations are given equal weight. The denominator in the above expression converges to $1/(1 - \lambda)$, so that the variance calculated in this method for N, which aspires to infinity, for day T also depends on yesterday's variance: $\sigma_T^2 = (1-\lambda)\sum_{t=0}^{\infty} \lambda^{t-1} r_{T-1} = (1-\lambda) r_T + \lambda \sigma_{T-1}^2$. In effect, this characteristic is widespread in the parameters of the market of variables in clusters: a shock to a specific parameter will be accompanied by additional shocks in the same time environment (Jackson et al., 1997). As for the decay parameter, the smaller it is, the greater is the weight of the latest observations. J.P. Morgan has set a decay parameter of 0.94 in its data base. Below we examine changes in the values given to risk in the representative trading book for two values, 0.97 and 0.94. Note that the problem of non-stationary variance has been treated extensively, and the solutions given for heteroskedacity are of the GARCH (1,1) kind, or those found in similar but more sophisticated models.[5]

Monte Carlo simulation

The method by which the Monte Carlo simulation is used to calculate VaR is non-parametric, and is considered to be the most sophisticated of the three methods.

The simulation is based on identifying the outstanding risk factors (which exert considerable influence on the trading book), and on constructing a joint distribution for those factors on the basis of past data. Subsequently, a large number of observations are sampled from the same distribution, and the profit or loss in the portfolio for each observation is calculated. The results are ranged in order—from the greatest loss to the largest profit—as in the other methods, and the value of the percentile (1 percent) that reflects the maximum loss that could be incurred at 99 percent probability is the VaR. In

[5] The treatment of the problem of the non-stationarity of changes in risk factors is beyond the scope of this paper.

the present paper VaR has been calculated using this approach, as follows: averages and variance-covariance matrices were calculated for changes in the risk factors (on the basis of past data), and some 10,000 observations of random changes were sampled on the basis of the assumption of a multi-normal distribution of the changes in the risk factors.

This method has several important advantages: first, in order to perform the simulation no assumptions need to be made for any specific model about the link betweeen the value of the trading book and changes in the risk factors, or regarding the distribution of changes in risk factors. Secondly, it makes it possible to deal with non-linear financial instruments, such as options, and it is also relatively easy to examine the effect of the stress tests on the value of the trading book. In addition, it is possible to use the simulation to monitor the process by which the value of the trading book is obtained as a function of the risk factors, in contrast with the other methods, in which only a final result is obtained. Furthermore, the initial results obtained in the other methods can be used as input for the Monte Carlo simulation. For example, it is possible to assess the distribution of the risk factors by means of the historical simulation, and to gauge the significance of the various risk factors (via their effect on the value of the trading book) and their interaction by means of the joint variance-covariance matrix.

The main disadvantage of this method is its slowness—the convergence of the Monte Carlo simulation to a 'real value' is of the order of $1/\sqrt{N}$, where N is the number of trajectories set. This means that in order to increase the level of accuracy by a factor of 10, it is necessary to run another 100 simulations. Thus, for example, a simulation containing 10,000 runs (trajectories) takes about 10 hours on a PC (Pentium 166 MHz). Note, however, that techniques for making the simulation more efficient have been developed; these reduce the variances within the trading book, and hence the number of runs required. These techniques are based on the known characteristics of the trading book, such as the links between the various risk factors. Another technique for speeding up the simulation run is to compress the trading book, thereby creating an artificial financial instrument whose risk characteristics are identical to those of the original trading book, and whose behavior it simulates.

3. CALCULATING CHANGES IN THE RISK FACTORS

The method of calculating periodical changes in the risk factors is very important, as it exerts considerable influence on the value of the trading book, and hence on VaR. Very little has been written in the literature about the way of calculating the daily changes in risk factors, and these have generally been assessed as rates of change. The three main ways of calculating the price changes are the additive, multiplicative, and mixed approaches. The advantages and disadvantages of each one will be discussed below, as will the way they are applied in the framework of the various ways of calculating VaR. The results, and their significance, will be reviewed in the section on empirical estimations.

The additive approach

Under the additive approach, the changes in risk factors are calculated by adding the difference between risk prices on day $j + 1$ and those on day j to today's market vector, P_t.

This method, which assumes that the variance of the risk factor does not depend on its level, is more suited for calculating changes in interest rates, especially when their absolute level is low. For example, if the interest rate rises from 4 to 5 percent, using this approach it will be translated into a 1 percentage-point change, while according to the multiplicative approach the change will be 25 percent of the interest rate.

The additive change in the prices of risk factors will therefore be calculated as follows:

(4) $$\Delta_{i,\tau}^{A} = x_{i,\tau} - x_{i,(\tau-1)},$$

where $x_{i,\tau}$ is the market value of the i risk factor on day τ. The expected value of the trading book on the basis of the daily changes will be:

(5) $$P_t\left(X_t + \Delta_{\tau}^{A}\right),$$

where X_t is the vector of prices of risk factors on the day of calculation, and Δ_τ the vector of additive changes on day τ. In the framework of the historical simulation, the change in each of the risk factors is first calculated as the difference between the parameters (the prices of the risk factors), i.e., N vectors of type Δ_τ (equal to the number of days in the estimation period—the window size) are obtained. After that, the profit or loss obtained as a result of those changes is calculated. The maximum loss obtained at a given probability (at a 99 percent confidence level) will be the VaR. The application of the additive approach in the framework of the variance-covariance matrix was presented in the previous section.

The multiplicative approach

Many economic variables, such as exchange rates, indices, and share prices, are characterized by multiplicative changes, pointing to the relative variance of the changes in those variables (risk factors). The calculation using this approach is obtained by multiplying the market vector today, P_t, by the ratio of the values of risk factors on day $j + 1$ to those on day j. According to the multiplicative approach, there is a correlation between the variance and the level of the variable (risk factor): the higher the level the greater the variance, and vice versa. This approach is better suited for calculating changes in exchange rates and share-price indices.

The multiplicative change in the prices of risk factors will be calculated on the basis of the following equation:

(6) $$\Delta_{i,\tau}^{M} = \frac{x_{i,\tau} - x_{i,(\tau-1)}}{x_{i,(\tau-1)}} = \frac{\Delta_{i,\tau}^{A}}{x_{i,(\tau-1)}} ,$$

where $x_{i,\tau}$ is, as stated, the value of the i risk factor on day τ, and the superscripts A and M denote the additive and multiplicative calculations respectively. The expected value of the portfolio on the basis of the daily changes will be:

(7) $$P_t\left(X_t \cdot \left(1 + \Delta_{\tau}^{M}\right)\right),$$

where X_t is the vector of market values of risk factors on the day of the calculation, and Δ^M_τ is the vector of the multiplicative changes on day τ.

The application of this approach within the framework of the historical simulation is as follows: the rate of change of each risk factor is calculated, and then the maximum loss (with a given probability) that may be incurred as a result of past changes is calculated. This is done in a way similar to that used for the historical simulation in the additive calculation. The application of the multiplicative approach in the framework of the variance-covariance matrix is more complex, however, as assumptions are made about the distribution of the changes in the risk factors.

We denote the risk factors by the vector $X = (x_1, x_2,..., x_n)$, where the changes are calculated as rates of change, i.e., $(x^1 - x^0)/x^0$, and we assume that the changes are distributed normally.

By multiplying and dividing the right-hand side of equation (1) (reflecting the changes calculated as rates of change) by x^0, we get the change in the value of the portfolio as a function of the rates of change of the risk factors:

(8) $$\mathrm{P}(\mathrm{X}^1) - \mathrm{P}(\mathrm{X}^0) = \mathrm{P}'(\mathrm{X}^0)\cdot(\mathrm{X}^1 - \mathrm{X}^0)\frac{\mathrm{X}^0}{\mathrm{X}^0} = \sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\mathrm{P}'_\mathrm{i}(\mathrm{X}^0)\cdot \mathrm{X}^0_\mathrm{i}\,\frac{\mathrm{X}^1_\mathrm{i} - \mathrm{X}^0_\mathrm{i}}{\mathrm{X}^0_\mathrm{i}}\right),$$

i.e., by using first-order derivatives (an approximation by the Taylor series), we obtain the value of the portfolio as a linear function of the rates of change with the weights $\left[\partial P\left(X^0\right)/\partial X_i\right]\cdot X_i^0$. As the rates of change of the risk factors are distributed normally, the distribution of the daily changes in the value of the portfolio will also be normal, with the following mean and variance:

(9) $$E\left[\frac{P\left(X^1\right) - P\left(X^0\right)}{P\left(X^0\right)}\right] = \sum_{i=1}^{n}\frac{\partial P\left(X^0\right)}{\partial X_i}X_i^0\mu_x$$

$$var\left[\frac{P\left(X^1\right) - P\left(X^0\right)}{P\left(X^0\right)}\right] = P'\left(X^0\right)X^0\cdot S_x\cdot X^{0T}P'^T\left(X^0\right),$$

where μ_x and S_x are, as stated, the vector of means and the variance-covariance matrix of the rates of change of the risk factors respectively. In other words, one percentile of the distribution of losses and gains in the portfolio, reflecting the maximum loss with a probability of 99 percent, can be written as:

(10) $$\mathrm{VaR}_{1\%}\left(\frac{P_1 - P_0}{P_0}, X_0\right) = \mu - 2.33\cdot\sigma.$$

The mixed approach

In this method the risk factors are divided into two groups: those, such as interest rates, changes in which are best captured by the additive approach, and those, such as exchange rates, changes in which are best captured by the multiplicative approach. Like the other two approaches, the simplest application of the mixed approach is for the historical simulation, where the framework of dealing with additive instruments is additive, and that for dealing with multiplicative instruments is multiplicative. For example, a change in the exchange rate will be calculated as the rate of change $(x^1 - x^0)/x^0$, and that in the interest rate will be calculated as the difference between them $(x^1 - x^0)$.

In order to apply the mixed approach for the variance-covariance matrix method, we transformed some of the changes, making them all either additive or multiplicative. This made it possible to calculate VaR by either the additive or the multiplicative approach, taking the transformation into account, on the basis of equations (4) and (10) above.

4. APPLYING THE VaR METHOD TO ISRAEL'S BANKING SYSTEM

Description of the trading book and the risk factors

In order to construct an internal model that will meet all the requirements of the Basle Committee as well as fitting the characteristics of the Israeli economy, we chose a representative trading book of a bank in Israel, comprising: Treasury bills of varying terms to maturity, ranging from two months to one year; government bonds that are indexed to the CPI (*Sagi*, *Galil*, and enhanced *Galil*) for redemption between one and seven years; US government T-bills with maturities of between one month and a year; dollar-indexed government bonds (*Gilboa*), for periods of between three months and two years; *Ma'of* (the 25 major shares traded on the TASE) shares; and shekel/dollar exchange-rate call options. The proportions of the investment in each financial instrument in the portfolio were selected in such a way as to reflect the representative trading book of a large commercial bank in Israel.

The second stage, after selecting the trading book, is to map the risk factors inherent in it. Several main risk factors influence the value of the book we have chosen: inflation, exchange rates (against the dollar, the mark, etc.), share-price indices (general, Mishtanim, *Ma'of*), the various yields to maturity (term structure) in the unindexed segment (for Treasury bills), the various yields to maturity in the indexed segment (for CPI-indexed government bonds), and the various yields to maturity in the foreign-exchange segment (dollar-Libor rate). Altogether we identified 22 risk factors in the portfolio.

With regard to interest-rate risk, there are in effect many risk factors that are derived from the term structure of interest rates, since the volatility of interest rates for one period is not identical with that of those for another one. J.P. Morgan has overcome the problem of multiple risk factors by defining a relatively small number of risk factors (representative rates) and dividing the flows in accordance with the periods set for them. We chose to create a continuous term structure for each indexation segment by linear interpolation between representative rates, and to ascribe each flow to the relevant risk factor as indicated by the time remaining to its redemption. Take, for example, the calculation of risk for a cash flow of 100 to be received in one and a half years from today, given only two risk factors—interest for a year and interest for two years: one way (as used by J.P. Morgan) resolves the problem by converting the flow for a year and a half into two income payments of 50, one for one year and the other for two years. The other approach is, as stated, to calculate the interest for a year and a half by linear interpolation of interest for a year and interest for two years.

The data base comprises daily changes for the 22 risk factors[6] derived from the chosen portfolio,[7] for the period between January 1994 and June 1997. We calculated the daily changes in the risk factors (prices) using the three methods described earlier—additive, multiplicative, and mixed; in the mixed method we took two rates of change (multiplicative method) for risk factors other than interest rates, e.g., exchange and inflation rates, share-price indices, and differentials (additive method) for interest rates. This is because calculating the simple rate of change is based on the assumption that the assets are similar in essence to consol-type bonds, although in actual fact the duration of the bonds in the banks' trading book differs from that of consols, so that the additive method is more appropriate.

Table 1 gives selected statistical data on the daily changes in each risk factor. The data show that the distribution of changes is not normal, as they display kurtosis as well as fat tails (skewness $\neq 0$). We also ran a Jarque-Bera test for each distribution, and the null hypothesis—that the variables are normally distributed—was rejected.

Table 1
Statistical Data on Daily Changes in Risk Factors

(percent)

	Min.	Med.	Max.	Avg.	STD	Skew.	Kurt.
Risk factors							
Inflation[a]	–0.004	0.044	0.103	0.042	0.023	0.466	3.494
Exchange rate[b]	–1.105	0.000	1.589	0.022	0.302	0.716	6.235
Share-price index[c]	–9.657	0.065	7.372	0.033	1.624	–0.001	6.180
Interest rates							
Nominal[d]	–1.940	0.000	1.830	0.003	0.288	–0.483	10.86
Real[e]	–0.600	0.000	0.600	0.002	0.112	–0.379	8.927
Dollar (indexed)[f]	–0.313	0.000	0.250	0.003	0.044	0.161	12.22
Dollar (denominated)[g]	–0.279	0.000	0.261	0.002	0.058	0.174	6.346

[a] Changes in CPI calculated on daily basis by number of trading days in month (linear interpolation). This method is used for ease of calculation; more precise methods can be used instead, however.
[b] Daily changes in NIS/dollar exchange rate.
[c] Daily changes in *Ma'of* Share-Price Index.
[d] Daily changes (percentage points) in yield to maturity on 3-month Treasury bills.
[e] Daily changes (percentage points) in yield to maturity on 3-year CPI-indexed bonds.
[f] Daily changes (percentage points) in yield to maturity on 3-month dollar-indexed bonds.
[g] Daily changes (percentage points) in yield to maturity on 1-year Treasury bills.

[6] These include, *inter alia*, the CPI; the NIS/dollar exchange rate; the *Ma'of* Share-Price Index; the nominal yield to maturity on T-bills for periods of one, three, six, and twelve months; the real yield to maturity on government bonds for periods of one, three, five, and ten years; the dollar-indexed yield to maturity on government bonds for periods of one, three, six, and twelve months; the dollar-denominated yield to maturity on government bonds for periods of three, six, and twelve months.

[7] Risk factors can be added or subtracted, in accordance with the portfolio selected.

Comparison of the standard and internal models

Table 2 gives the results of the calculation of the capital requirement for market risks (by means of an internal model) obtained from calculating VaR by the three methods (historical simulation, variance-covariance matrix, and Monte Carlo simulation). For purposes of comparison, the standard capital requirement, calculated in accordance with the instructions of the Basle Committee of 1996, is also presented. The capital requirements were calculated, as stated, for the year ending in May 1997 for the representative trading book of a bank in Israel (we use 'trading book' and 'portfolio' interchangeably).

The most salient finding is that the standard capital requirement—about 6 percent of the value of our portfolio—is significantly higher than the capital requirements obtained from the three internal models, which vary from 1.7 to 2.8 percent of the value of the portfolio (Figure 1 and Table 2). This finding was expected, as the standard capital requirement is based on a conservative and rough estimate, reflecting the attitude of the authorities to dealing with risks. Moreover, the capital requirements set on the basis of the standard approach are far more volatile; this is because this approach does not take the correlations between the various assets into account, and because there is no moving average of the capital requirements (in internal models a moving average is required for the 60 days preceding the day of the calculation), which contributes to the stability of the capital requirements.

Both the table and the figure show that there are no great differences between the capital requirements based on the various methods of calculating VaR. Note, in this connection, that some of the risk factors (e.g., the inflation rate) have an average of positive changes and that the position of the trading book of banks in Israel is long; these facts affect the capital requirements derived from the different ways of calculating VaR.

Table 2
Capital Requirements Against Market Risk Calculated in Three Different Ways, and the Standard Approach[a]

	Standard approach	Historical simulation	Var-covar matrix	Monte Carlo simulation
Minimum	5.96	2.39	2.42	2.49
Median	6.64	2.64	2.66	2.58
Maximum	7.69	2.74	2.89	2.76
Average	6.67	2.60	2.63	2.59
STD	0.40	0.11	0.11	0.09

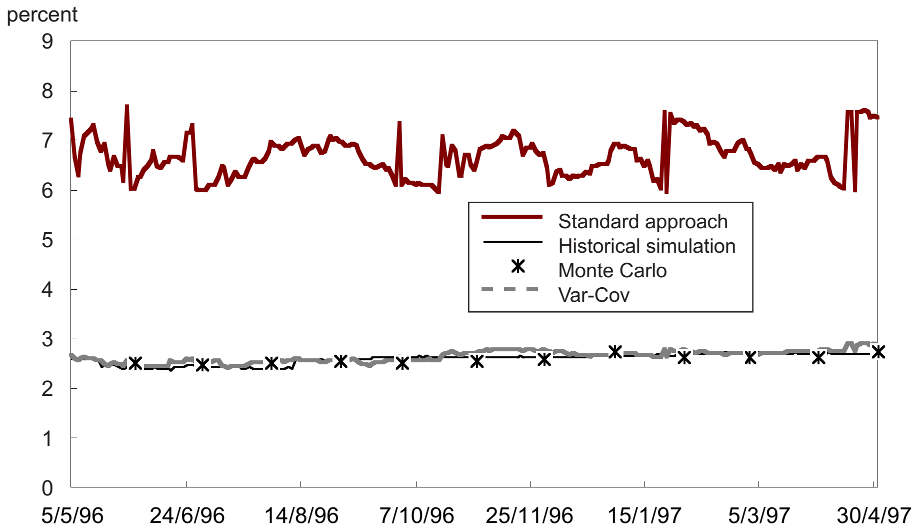
[a] May 1996 to May 1997, proportion of value of trading book after incorporating supervisory factor.

1. The calculations are based on a database of daily data for 300 holding days, the last of which is 29.6.1997.

2. The capital requirement obtained from the internal models according to the three techniques was calculated in accordance with the recommendations of the Basle Committee of January 1996. The holding period is 10 trading days (daily changes were multiplied by a factor of $\sqrt{10}$), and the confidence level is 99 percent. A supervisory conversion factor of 3 was also used (according to the Basle Committee, a factor of between 3 and 4 can be used).

3. The data for the 10,000 times Monte Carlo simulation are based on only 12 monthly observations.

Figure 1
Capital Requirements According to the Standard Aproach and Various Methods for Calculating VaR

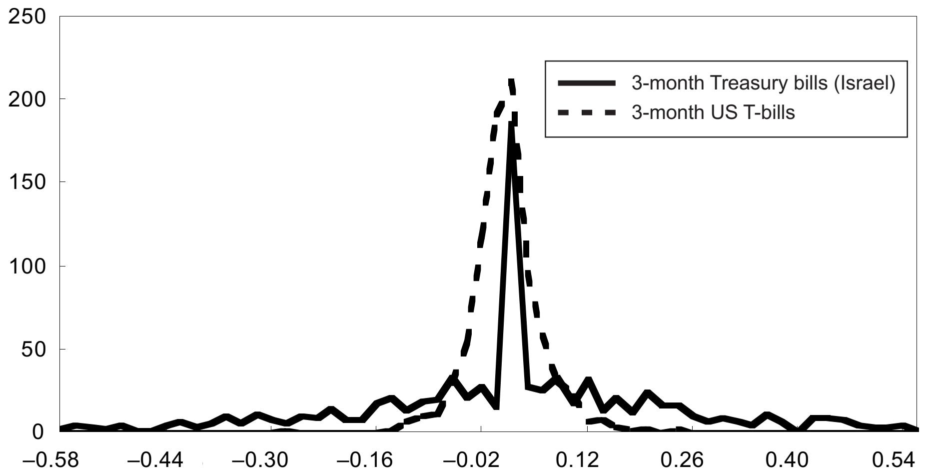


Economic characteristics unique to Israel

Several economic characteristics that are unique to Israel must be taken into account in building an internal VaR model for estimating market risks. The three main factors distinguishing Israel from western economies are: 1. A high level of inflation and the existence of indexation arrangements; 2. An exchange-rate regime involving a band with a positive slope and a basket of currencies (as opposed to the floating exchange rate customary in the west); 3. A thin capital market (shares and bonds with low liquidity). The influence of these features on VaR is non-negligible, so that if one of them is ignored, the estimate of VaR will be biased. In addition, the assumption underlying the method using the variance-covariance matrix is that the changes in risk factors are normally distributed. Applying the method to Israel could lead to biased results, as some of these changes are not normally distributed. The outstanding example of this is the distribution of changes in the exchange rate when the actual rate is near one of the limits of the band.

Figure 2 illustrates the differences between daily changes in the yield on Israel's Treasury bills and those on US T-bills. It is quite clear that the averages are similar (around zero), but the dispersion around the average of Treasury bills is greater than that of the US equivalent, regardless of the absolute values of interest rates. It is to be expected that the VaR of the former will consequently be higher than that of the latter (assuming that this is

Figure 2
Distribution of Daily Changes in Rates of Yield to Maturity, January 1994–June 1997



the only asset in the portfolio).

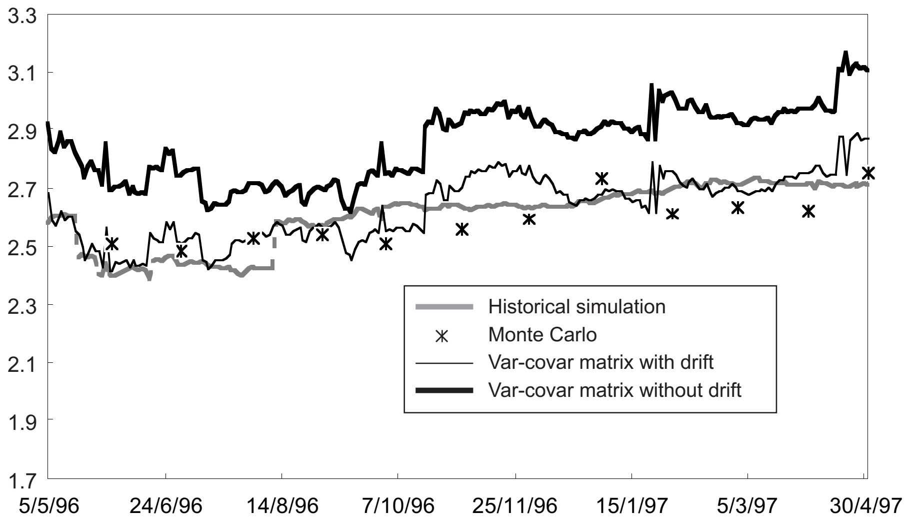
Comparison of the various methods of calculating VaR

An important finding for Israel's banking system is that there is a marked difference between the capital requirements derived from the variance-covariance matrix method, assuming the averages of the changes in risk factors equal zero, and those derived from the same method without this assumption. The explanation for this is that the distributions of the changes in some of the risk factors are not normal if the average is zero, as is assumed in the variance-covariance matrix approach, but are characterized by a more humped structure (kurtosis > 3) and positive assymetry (skewness > 0). In view of the findings (Table 1), it would appear that in applying this technique to financial institutions in Israel it cannot be assumed that the average of the changes in risk factors equals zero.

Figure 3 shows the three most widely used methods for calculating VaR. The VaR is highest when the variance-covariance matrix method is used and no account is made for drift, i.e., according to the method used abroad (e.g., RiskMetrics). The other methods, including the variance-covariance matrix method taking drift into account, yield similar results. The conclusion to be drawn from this is that when this approach is applied to Israel account must be made for drift (which is positive, i.e., reduces the capital requirement).

In addition to the level of exactness, in ascertaining the appropriate method for calculating VaR it is also necessary to take the cost of constructing and maintaining a model according to each method into account. For example, calculating VaR for a given day (using a 166 MHz PC) takes 200, 1,000, and 5,000 seconds for the variance-covariance matrix method, historical

Figure 3
Capital Requirements According to the Three Methods of Calculating VaR in the Framework of Internal Models
(percent of portfolio)



simulation, and Monte Carlo simulation respectively. We ran the Monte Carlo simulation several times, each time increasing the number of trajectories, and obtained a process of convergence for a specific value (by the end of the process there were 10,000 trajectories on each run).

Comparison of ways of calculating changes in risk factors

We examined the effect of the ways of calculating changes (additive, multiplicative, and mixed) on VaR. Table 3 and Figure 4 give the effect of the method of calculation on VaR using

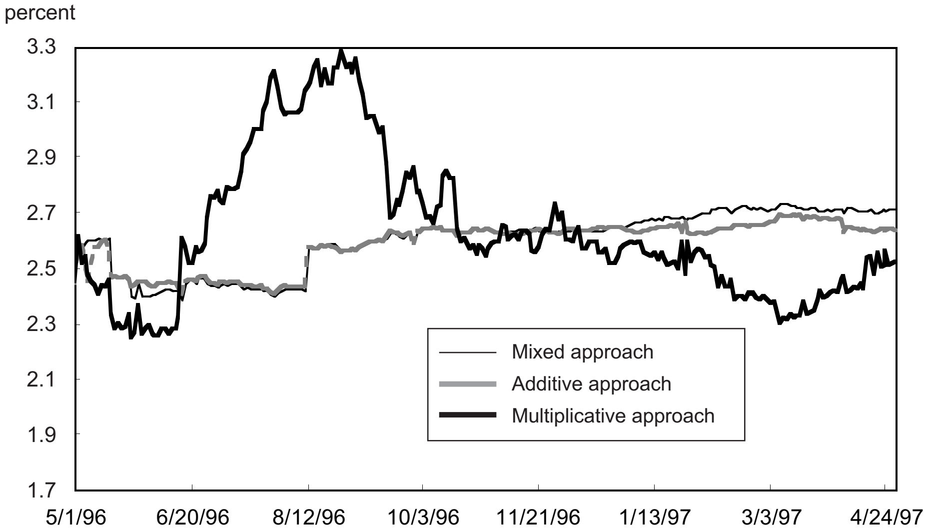
Table 3
Capital Requirements Against Market Risk Using the Historical Simulation and Variance-Covariance Methods, According to Three Ways of Calculating the Changes in Prices of Risk Factors[a]

	Historical simulation			Var-Covar matrix		
	Additive	Multiplicative	Mixed	Additive	Multiplicative	Mixed
Minimum	2.40	2.24	2.39	2.39	2.31	2.42
Median	2.63	2.57	2.64	2.64	2.66	2.66
Maximum	2.69	3.29	2.74	2.83	3.07	2.89
Average	2.59	2.63	2.60	2.62	2.66	2.63
STD	0.09	0.26	0.11	0.10	0.18	0.11

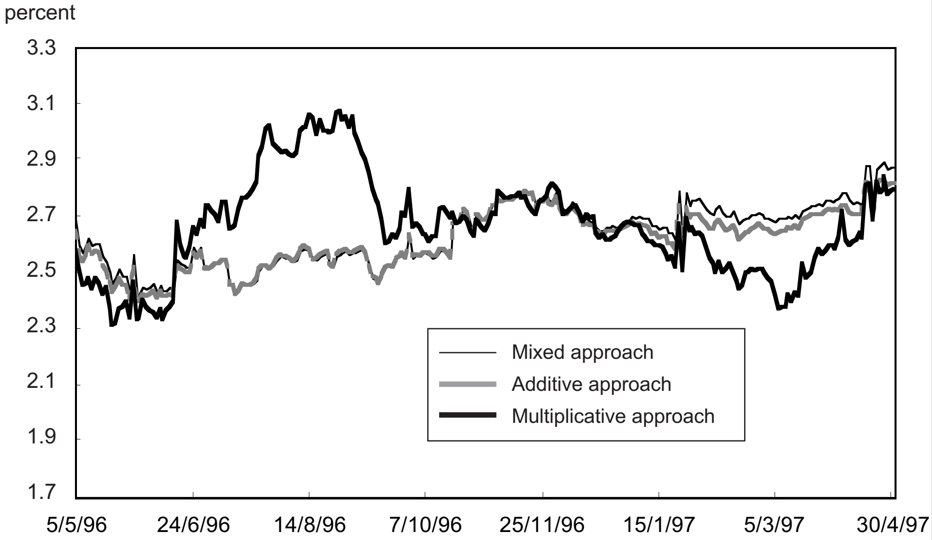
[a] May 1996 to May 1997, proportion of value of portfolio after incorporating supervisory factor.

Figure 4
Capital Requirements According to Three Ways of Calculating Changes in Prices of Risk Factors
(percent of portfolio)

A. Historical simulation



B. Var-covar matrix



the historical simulation and variance-covariance approaches.

As the table and the figure show, the method of calculation has a marked effect on the result; the multiplicative approach is far more volatile, with a larger standard deviation and average change, irrespective of the approach chosen. Consequently, when the drift is positive (the prices of risk factors rise) the capital requirement derived from the multiplicative method is higher (the left-hand part of Figure 4), and when the drift is negative the reverse is the case (the right-hand part of Figure 4). The additive method is similar to the mixed method, indicating that the risk factors associated with interest rates are more significant than those connected with exchange rates or price indices, a factor which is generally dependent *inter alia* on the portfolio selected.

Our conclusion is that for a typical trading book, the method of calculating the daily changes (multiplicative vis-à-vis additive and mixed) is no less—and possibly even more—important than the approach used (historical, var-covar, or Monte Carlo simulation).

Calculating daily changes versus changes during 10 business days

The Basle Committee determined two ways of calculating VaR using an internal model for a holding period of 10 business days: the first involves calculating the daily changes in risk factors, and multiplying the final result, i.e., the VaR, by $\sqrt{10}$; while the second involves the changes in risk factors during ten business days. In the present study we

Table 4
Capital Requirements Against Market Risk According to Three Methods of Calculating VaR[a]

1. On the Basis of Daily Changes in Risk Factors[b]

	Additive		Multiplicative		Mixed	
Confidence level (%)	95	99	95	99	95	99
Historical simulation	1.74	2.62	1.75	2.38	1.88	2.72
Var-covar matrix	1.81	2.68	1.71	2.54	1.86	2.75
Monte Carlo[c]	1.74	2.49	1.66	2.42	1.78	2.60
Standard approach			6.11			

2. On the Basis of Changes Over 10 Trading Days[d]

	Additive		Multiplicative		Mixed	
Confidence level (%)	95	99	95	99	95	99
Historical simulation	2.14	3.23	2.18	2.97	2.33	3.36
Var-covar matrix	1.82	2.90	1.72	2.78	1.90	3.04
Monte Carlo[c]	1.76	2.59	1.67	2.72	1.81	2.89

[a] For 29.6.1997; percentage of value of portfolio after incorporating supervisory factor.

[b] The calculations are based on a database of daily data for 300 holding days, the last of which is 29.6.1997.

[c] The capital requirement obtained from the internal models according to the three techniques was calculated in accordance with the recommendations of the Basle Committee of January 1996. The holding period is 10 trading days (daily changes were multiplied by a factor of 10), and the confidence level is 99 percent. A supervisory conversion factor of 3 was also used (according to the Basle Committee, a factor of between 3 and 4 can be used).

[d] The data for the 10,000 times Monte Carlo simulation are based on only 12 monthly observations.

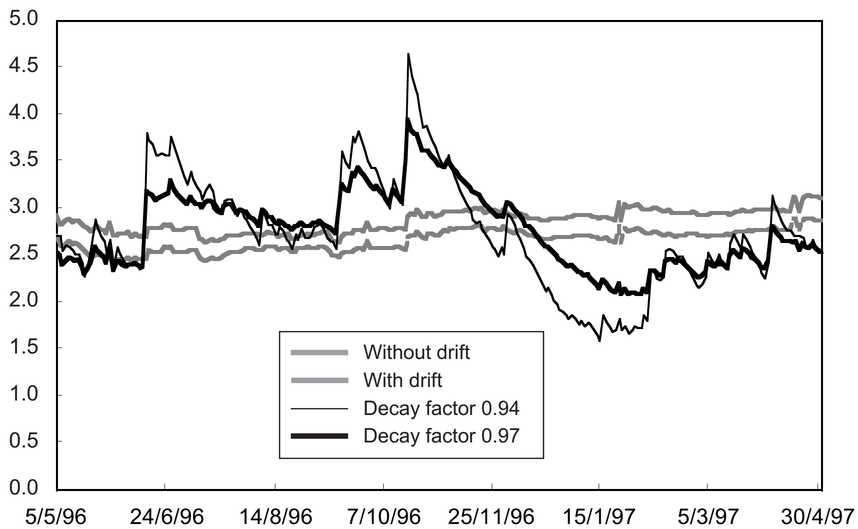
calculated VaR by both these methods and compared them (Table 4).

The findings show that when VaR is calculated on the basis of ten business days it is higher than when it is based on daily changes. This result is obtained for the three approaches to calculating VaR, whether the confidence level is 99 percent or 95 percent, and is particularly prominent in the historical simulation. Consequently, our conclusion is that here, too, it is necessary to calculate the changes for ten business days rather than assuming that the holding period is one day and multiplying it by $\sqrt{10}$, or alternatively one can measure drift and volatility separately, and scale them appropriately.

The decay factor in the variance-covariance matrix approach

As indicated above, the decay factor plays an important role in applying the variance-covariance matrix method, because when the weight of the observations is uniform the results obtained may be dependent on the distant past, and hence no longer relevant. In this study we examined two decay factors, 0.94 (similar to RiskMetrics) and 0.97, the smaller of which gives greater weight to the recent past. Figure 5 presents the capital requirements according to the variance-covariance matrix method without a decay factor ($\lambda = 1$), and with decay factors of $\lambda = 0.97$ and $\lambda = 0.94$. As can be expected, when the decay factor is larger (or when there is no decay factor at all) the capital requirements are less volatile. No exceptional events occurred in the sample period (May 1996 to May 1997), so that the (equal) weight of the early observations served as a moderating factor. With a small decay factor, on the other hand, greater weight is ascribed to the recent observations, so that changes in the recent past are fully expressed.

Figure 5
Capital Requirements According to Var-Covar Matrices and Selected Decay Factors
(percent of portfolio)



Backtesting the models

The next stage after obtaining the results is to check their stability and reliability over time. The Basle Committee recommended using the number of exceptions as a criterion for backtesting the internal models, i.e., the situations in which the actual loss in the portfolio exceeded the calculated VaR during a given period. The Committee defined three zones: the green zone, a situation in which the number of exceptions during one year is 4 or less, indicating that the model is stable, so that the capital requirement is the VaR multiplied by the supervisory scaling factor, which is 3; the yellow zone, where the number of exceptions in one year is between 5 and 9, in which case the capital requirement will be the VaR multiplied by the supervisory scaling factor, which ranges from 3 to 4 according to the decision of the supervisory authority; and the red zone, where the number of exceptions during one year is 10 or more, in which case the model is defined as inadequate.

In order to undertake backtesting of the various models we calculated the values of VaR for each business day during the year ending at end-June 1997, using a window size of 300 business days. We then compared the values of VaR obtained with the actual losses, and counted the number of exceptions. As Figure 6 shows, there is one exception according to all the models; the findings therefore indicate good adequacy (green zone). In the Monte Carlo simulation we examined only 12 observations for the last year because of the considerable time it takes to run each simulation. Nonetheless, several upward exceptions were obtained (beyond what the models had predicted).

Note that during the period reviewed, the actual loss never exceeded the capital required (VaR • 3).

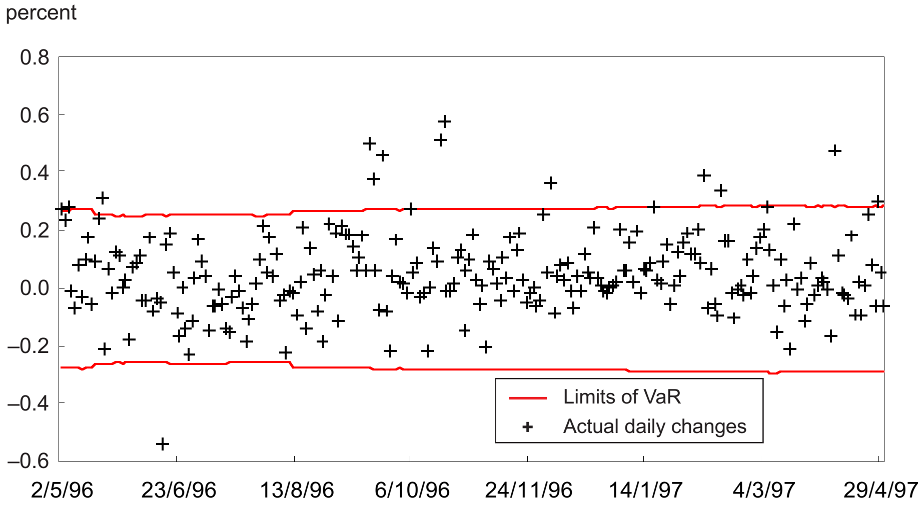
Portfolio selection and capital requirements

In the framework of our study, we also reviewed several types of trading books. In addition to the original portfolio we constructed four portfolios with the following structures: one with a significant proportion (about 60 percent) of unindexed assets, the second with 70 percent of CPI-indexed assets, the third with 65 percent of assets indexed to foreign currency, and the fourth with 20 percent in stocks. These portfolios were chosen to represent different types of Israeli banks at the end of 1997 (some effects were intentionally enlarged). The following table shows the capital requirements for all these portfolios relative to the standard approach.

As Table 5 shows, the portfolio mix is very important in defining riskiness and the corresponding capital requirements. The standard approach leads to the lowest capital requirements for the CPI-indexed portfolio, since the basic currency is the CPI-indexed shekel. The portfolios have different risk exposures and cannot be compared directly, but the general observation is that for a diversified portfolio an internal model has a significant advantage over the standard approach in capital requirements.

Figure 6
Backtesting of Internal Model
(percent of portfolio, without root multiplication, with control coefficient)

A. Historical simulation



B. Var-covar matrix

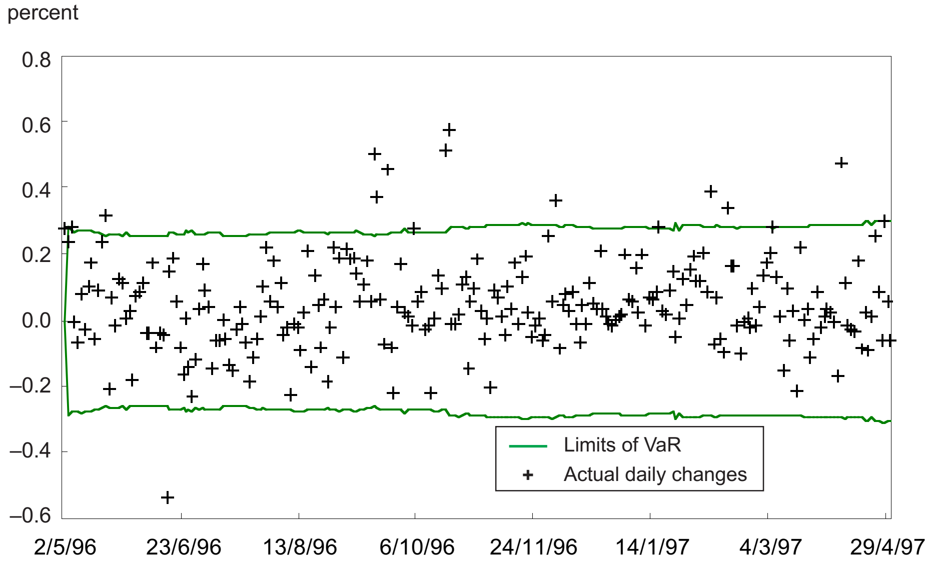


Table 5a
Different Types of Portfolio Chosen

(percent)

	Shares	For-Ex	Indexed	Unindexed	Diversified	Total
Treasury bills	18.7	14.8	14.0	**57.6**	22.6	23.2
Galil	36.2	19.1	**69.0**	22.2	45.3	44.0
Gilboa and T-bills	25.4	**64.1**	15.1	18.1	30.4	32.8
Shares	**20.3**	2.4	2.4	2.6	2.2	
Stock options	–0.5	–0.5	–0.5	–0.5	–0.4	

Table 5b
Capital Requirements Against Market Risks, by the Three Methods and Type of Portfolio (averages)[a]

(percent)

	Shares	For-Ex	Indexed	Unindexed	Diversified
Monte Carlo simulation	4.45	4.00	3.34	2.17	2.59
Var-covar matrix	4.55	3.69	3.37	2.16	2.63
Historical simulation	5.52	4.09	3.67	2.76	2.60

[a] Size of window: May 1996 to May 1997; percent of value of portfolio.

Table 5c
Standard Deviation of Capital Requirements Against Market Risks, by the Three Methods and Type of Portfolio[a]

(percent)

	Shares	For-Ex	Indexed	Unindexed	Diversified
Standard approach	**0.64**	**0.60**	**0.60**	**0.62**	**0.40**
Monte Carlo simulation	0.93	0.24	0.11	0.17	0.11
Var-covar matrix	0.89	0.18	0.14	0.17	0.09
Historical simulation	1.31	0.15	0.24	0.39	0.11

[a] Size of window: May 1996 to May 1997; percent of value of portfolio.

5. SUMMARY AND CONCLUSIONS

In this study we applied three widely-used methods of calculating VaR used in internal models in Israel—the historical simulation method, the variance-covariance matrix method, and the Monte Carlo method—and compared them with the standard approach of the Basle Committee (1996). In a test during one calendar year we found that the capital requirements on a typical trading book of a large Israeli bank were significantly higher and more volatile than those calculated on the basis of any of the three methods. This means that for the banks in Israel (or at least the large ones) it is worth adopting these methods in the framework of internal models rather than the standard approach, which requires pinning down a larger amount of more volatile capital against market risks.

An analysis of the risk factors in Israel indicates that daily changes in them are not normally distributed, *inter alia* because of features that are unique to Israel: high inflation and a system of indexation, a narrow market, and an exchange-rate regime conducted within a band. Consequently, the assumption underlying the variance-covariance matrix method, which has been adopted all over the world (e.g., RiskMetrics™), is not appropriate for Israel. We found that the capital requirements derived from the historical and Monte Carlo simulations are smaller than those calculated on the basis of the variance-covariance matrix method that is widely used abroad. However, when account is made for drift, which was found to be positive in the sample period, the capital requirements obtained in all three methods were very similar. We also found that portfolio structure is important; in all the implementations the internal models have a significant advantage over the standard approach for well-diversified portfolios.

Other subjects associated with the instructions issued by the Basle Committee (1996) and the application of the various methods of calculating VaR were examined on the basis of a typical portfolio and risk factors in Israel. As expected, the volatility of VaR rose as the decay factor in the variance-covariance matrix method decreased; VaR based on changes during ten days is greater than that based on daily changes; backtesting indicates that the methods and parameters selected are reasonable, and only one exception was found.

One of the subjects that the literature on methods of calculating VaR has tended to neglect is how to calculate daily changes. Almost all the research studies on the subject calculate the daily change rather than the relative change. We examined three ways of calculating this: multiplicative (rate of change), which is suitable mainly for share-price indices and exchange rates; additive (absolute change), which is suitable mainly for bonds, especially coupon bonds; and mixed, which uses both the first two approaches, in accordance with the character of the risk factor. Our tests indicate that the multiplicative approach is far more volatile than the other two, regardless of which of the three methods for calculating VaR is used. Our main conclusion is that choosing the appropriate way of calculating daily changes is at least as important—and possibly even more so—as choosing the method of calculating VaR.

The main points not decided unequivocally by the Basle Committee but which could

[8] Despite the freedom granted to the banks in choosing the assumptions and ways of calculation, the supervisory authorities check whether the models employed by the banks are in fact appropriate, by means of backtesting. This makes it possible to assess a bank's capital requirement against the market risks embodied in its trading book. In this study, we examined the effect of some of these points on VaR.

affect VaR, are as follows:[8]

1. The way VaR is calculated: apart from the three approaches discussed in this study—historical simulation, variance-covariance matrix, and Monte Carlo simulation—there are other ways of calculating this. The treatment and pricing of derivatives and similar instruments by means of various models may also yield different results.

2. Estimation period: the basic assumption according to the Basle Committee (1996) is that ten business days is a long enough period to rebalance the portfolio. However, the Committee permits VaR to be calculated on a daily basis and the results multiplied by $\sqrt{10}$. This implicitly assumes that the volatility over ten business days is greater by a factor of $\sqrt{10}$ than it is over one day. If this assumption does not hold, the results can differ significantly.

3. Confidence level: the Basle Committee recommends choosing a confidence level of 99 percent, compared with the 95 percent used by J.P. Morgan.

4. Sample window size: the minimum sample period required by the Basle Committee is one calendar year (250 business days). Jackson et al. (1997) found that enlarging the sample window stabilizes the VaR calculated, as the distribution of changes is based on more observations.

5. Allocating maturity periods by 'buckets' (intervals along the yield curve): the Basle Committee recommends breaking down future flows into at least six buckets, and determining a representative interest rate for each one. The purpose of this is to reflect interest-rate risk while taking the term structure of the yield curve into account. Choosing a larger number of buckets for each indexation segment makes it possible to attain greater precision in estimating risk. Another way of estimating interest-rate risk (the way adopted in this study) is to take into account the exact point at which the cash flows are received and transferred (in contrast with the division into buckets). The term structure of the yield curve is calculated by means of interest-rate interpolations.

6. The decay factor: as noted above, the smaller the decay factor, the greater the weight attributed to recent periods. Consequently, if the distribution of the changes in the variance-covariance matrix approach is very volatile, the choice of decay factor will have a marked effect on VaR.

7. Treatment of distribution characteristics: sophisticated internal models for calculating VaR take characteristics of the distribution of non-normal changes, such as fat tails, heteroskedacity, etc., into account.

8. Way of calculating changes: the way changes in the parameters of the market are calculated may also have a significant effect on the results. Most studies calculate the rate of change as the log of the parameter at time *t less* its value at time $t-1$. Nevertheless, as shown above, it is more appropriate to calculate the risk on coupon bonds on the basis of the changes in percentage points rather than as a rate of change.

In this study we tackled several topics connected with the application of internal models of VaR in Israel. Other topics which should be addressed in the future, include: 1. decomposing VaR into its components and examining its sensitivity to the various risk factors and the structure of the portfolio chosen; 2. examining the stationarity of the distribution of changes in risk factors, and undertaking stress testing; 3. the narrowness of Israel's financial markets; 4. an exchange-rate regime that is administered within a band. Referring to these points in the framework of the application of various methods of calculation should improve accuracy and reliability, so that appropriate and adequate capital is held against market risks.

APPENDIX: DIRECTIVES OF THE BASLE COMMITTEE (JANUARY 1996): THE STANDARD APPROACH AND INTERNAL MODELS

In January 1996 the Basle Committee issued a framework for estimating market risks and their capital requirements, and this was adopted by the western countries. Thus, since 1998 the G10 countries[1] have been required by their domestic supervisory authorities to estimate the market risks implicit in their activities and to maintain a level of capital adequacy that will cushion losses if these risks are realized. Market risks are defined as the risk of loss inherent in financial instruments, both on and off the balance sheet, as a result of changes in market prices (interest rates, exchange rates, share prices, etc.).

In this study we examine the application of the guidelines of the Basle Committee to Israel's banking system. The committee presented two basic ways of measuring market risks: the first, using the standardized approach set by the committee, is intrinsically coarser and more conservative; the second is by means of an internal model set by the bank, and is subject to the approval of the supervisory authorities.

Both the standardized and the internal models contain four risk categories, corresponding with the character of assets: bonds, shares, foreign currency, and commodities. Exposure to market risk in the bonds and shares categories relates solely to the bank's trading book, whereas exposure to the risks associated with intermediation in foreign currency and commodities relates to the entire banking book. Each risk category is divided into two parts, systemic risk, which is general market risk, and specific risk, which derives from the characteristics of each asset. Provided the supervisory authorities agree, the banks can include untraded assets or off-balance-sheet items in the trading book if they are used for hedging positions in it. In these cases, they are not included in the calculation of specific risk, but still require capital for counterparty risk.

Banks that are willing and able to use internal models for estimating market risk must meet certain qualitative and quantitative requirements.

The qualitative requirements include:

- An independent risk-management unit that is directly under the senior management and submits frequent reports on exposure to market risks and ways of managing and reducing it;
- Qualitative standards for internally supervising the use of risk-assessment models;
- A decision regarding factors constituting market risk;
- A verification procedure for backtesting the use of an internal model by bodies outside the bank, incorporating a comparison of the results of the model with actual *ex post* results;
- Guidelines for running stress tests which should represent extreme situations of stress in the financial and capital markets.

The quantitative criteria are as follows:

- VaR is calculated daily;
- A one-tail confidence interval at a confidence level of 99 percent is used in calculating VaR;

[1] Belgium, Canada, England, France, Germany, Holland, Italy, Japan, Luxembourg, Sweden, Switzerland, and the US.

- The calculation of VaR is based on the assumption of a minimal holding period of ten trading days;
- The database used in constructing the internal model includes complete data for at least one year;
- The database of the internal model is updated at least every three months;
- The model refers to all the substantial market risks to which the bank is exposed;
- The model includes the treatment of options, taking their non-linear nature into account.

The Basle Committee also determined that the capital requirement against market risks for a bank using an internal model will be the higher of the following two: 1. The bank's VaR for the previous business day; 2. The average of the bank's daily VaR for the previous 60 business days, multiplied by a supervisory scaling factor.

The minimum factor has been set at 3, and the domestic supervisory authority is entitled to add to it an amount between zero and one, in accordance with the quality of performance of the bank's internal model.

In accordance with the committee's recommendations, market risks can be calculated by means of an internal model in either of two ways: the first is to calculate exposure to each category of market risk (interest-rate, exchange-rate, share-prices, etc.) separately, and to add them up in order to obtain the total exposure index; this method, known as the *building block approach*, does not take the correlations between the various risk factors into consideration. The second approach, known as the *portfolio approach*, relates to the bank's total transactions in one investment portfolio, thereby taking these correlations into account.

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Value at risk (VaR) is a statistic that measures and quantifies the level of financial risk within a firm, portfolio, or position over a specific time frame.Â VaR modeling determines the potential for loss in the entity being assessed and the probability of occurrence for the defined loss. One measures VaR by assessing the amount of potential loss, the probability of occurrence for the amount of loss, and the timeframe. For example, a financial firm may determine an asset has a 3% one-month VaR of 2%, representing a 3% chance of the asset declining in value by 2% during the one-month time frame. The conversion of the 3% chance of occurrence to a daily ratio places the odds of a 2% loss at one day per month.