Spreadsheets in Education—The First 25 Years

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Abstract

Spreadsheets made their first appearance for personal computers in 1979 in the form of VisiCalc [45], an application designed to help with accounting tasks. Since that time, the diversity of applications of the spreadsheet program is evidenced by its continual reappearance in scholarly journals. Nowhere is its application becoming more marked than in the field of education. From primary to tertiary levels, the spreadsheet is gradually increasing in its importance as a tool for teaching and learning. By way of an introduction to the new electronic journal Spreadsheets in Education, the editors have compiled this overview of the use of spreadsheets in education. The aim is to provide a comprehensive bibliography and springboard from which others may develop their own applications and reports on educational applications of spreadsheets. For despite its rising popularity, the spreadsheet has still a long way to go before becoming a universal tool for teaching and learning, and many opportunities for its application have yet to be explored.

The basic paradigm of an array of rows-and-columns with automatic update and display of results has been extended with libraries of mathematical and statistical functions, versatile graphing and charting facilities, powerful add-ins such as Microsoft Excel’s Solver, attractive and highly-functional graphical user interfaces, and the ability to write custom code in languages such as Microsoft’s Visual Basic for Applications. It is difficult to believe that Bricklin, the original creator of VisiCalc could have imagined the modern form of the now ubiquitous spreadsheet program. But the basic idea of the electronic spreadsheet has stood the test of time; indeed it is nowadays an indispensable item of software, not only in business and in the home, but also in academe. This paper briefly examines the history of the spreadsheet, then goes on to give a survey of major books, papers and conference presentations over the past 25 years, all in the area of educational applications of spreadsheets.

Keywords: Spreadsheet, education, mathematics education, statistics education, survey.

1 A Brief History

Paper-based spreadsheets have been around for centuries, primarily for use as record-keeping and accounting tools. The first electronic spreadsheet, VisiCalc [45], [36] appeared in 1979, created by Dan Bricklin (concept) and Bob Frankston (programmer) for the Apple II platform. It was conceived and developed as a tool to do repetitive calculations for Bricklin’s studies at Harvard Business School. They formed a company, Software Arts, to market the product. In 1981, Bricklin received the Grace Murray Hopper Award from the Association for Computing Machinery (ACM) for the creation of VisiCalc. It has been said that VisiCalc was the application, more than any other that sold millions of Apple II computers. Houghton [87] notes:

The invention of the spreadsheet made personal computers have real value in the marketplace and legitimated the personal computer industry. Without the invention of this software category, spreadsheets, the impact of the personal computer might have been delayed for years.
Bricklin and Frankston sold the rights in VisiCalc to Lotus Development Corporation, which developed Lotus 1-2-3, a huge killer application for the new IBM PC in 1982. This product introduced rudimentary database and graphics functionality into the spreadsheet domain, and it dominated the market for most of the 1980s. Then came Microsoft Excel, which was developed first for the Apple Macintosh, but which was also the first real Microsoft Windows application. In fact, the earliest versions of Excel ran under MS-DOS, but with a special Windows runtime environment [193]. Other major spreadsheets were SuperCalc (1980, for CP/M operating system), Multiplan (Microsoft), PlanPerfect (WordPerfect Corp.), Quattro Pro (Borland), VP-PLANNER and AsEasyAs. Since the mid-1990s, Microsoft has held the dominant market share, and now commands in excess of 90% of the spreadsheet market [193].

The basic paradigm of an array of rows-and-columns with automatic update and display of results has been extended with libraries of mathematical and statistical functions, versatile graphing and charting facilities, powerful add-ins such as Microsoft® Excel’s Solver, attractive and highly-functional graphical user interfaces, and the ability to write custom code in languages such as Microsoft’s Visual Basic for Applications®.

Useful summaries of spreadsheet history are given by Power [154] and Walkenbach [194] and the authors gratefully acknowledge these sources in preparing the present very brief section on the historical background and emergence of electronic spreadsheets. Dan Bricklin’s own website [45] has some very interesting titbits of information and, at the time of writing, also includes a downloadable copy of VisiCalc. Bob Frankston’s website [71] is also interesting.

2 Programming vs Spreadsheets

As early as 1984, just one year after Lotus 1-2-3 made its presence felt in the commercial market, educators were beginning to discuss their experiences with using spreadsheets in education [25], [26]. Hsiao [88] makes the point that while computers are clearly useful tools for education generally, one of the main disadvantages is having to program them. In many cases, (at least in 1985), students had to learn a programming language in order to benefit from computers. Hsiao observes that use of spreadsheets helps to get around this problem. This view is still supported by other writers; for example, Morishita et al [133] state that:

Our experience in computing was that it took a very long time to learn computer languages and it was sometimes very hard to obtain proper results in a limited time. The spreadsheet, however, is rather easy to use and almost instantaneous numerical simulations are possible.

Hsiao considers examples such as function tabulation and plotting, integration by Simpson’s rule, and makes a brief mention of the possibility of matrix computations, including the Gauss-Jordan and inverse matrix methods of solving linear algebraic systems. Although this is quite an old paper, it is clear that the potential for educational uses of spreadsheets wherever mathematics is involved was obvious to many people even in the early 1980s. The author makes some interesting closing remarks.

1. The first is that the spreadsheet has allowed teachers to adopt a middle course, compared to the extremes of fully coding an algorithm in some programming language such as BASIC or Pascal, or using an off-the-shelf package with a canned solution. It is argued that neither of these methods is ideal for learning and the spreadsheet approach is recommended.1

2. The second one concerns the absence (in 1985) of a number of desirable features, such as transposition of rows and columns, absence of regression lines on scatter graphs, 3D graphs, and so on; all of these features are available in modern versions of spreadsheets such as Microsoft’s Excel.

1In many instances, at least, the present authors are in agreement.
Essentially the same points are made in a more recent paper by Steward [176]:

I would suggest that when both are possible, students find it easier and quicker to use a spreadsheet than write a computer program. Moreover, once written a program can often mask the mathematics that it is intended to represent, while on a spreadsheet the procedure is constantly exposed.

In this paper [176], the author considers three examples illustrating his claim: linear regression from first principles, convergence of recursive sequences, and first-order ordinary differential equations.

Reif and Almeida [157] give the example of the spreadsheet being used to (comprehensively) solve the Birthday Problem, which can be stated as finding the least number of people in a group for which there is a probability $\geq 0.5$ that two will have the same birthday. In comparing the cost/benefits of programming vs spreadsheets, they say:

...the prime need is for a medium which will facilitate consideration of conceptual issues while requiring minimal technical expertise, will provide insights into the mathematical context without necessitating attention to extraneous distractions, will permit modification without the need for major changes in design and will flag and encourage the pursuit of connected enquiries.

It is the spreadsheet that meets all of the above.

With access to VBA (Visual Basic for Applications), educators are also finding great benefit in tying programming to spreadsheet use, thereby overcoming much of the time spent in organizing data input and output. For example, Martin [121] outlines a course for Operations Research students in which he reports that:

...using the spreadsheet as a platform, the student is led to a position where they can write virtually a stand-alone program to support a simple OR application, even from a position of little previous spreadsheet or computing experience.

From the above, it appears that if the goal is not to teach a programming language, but to achieve understanding of a concept/topic for which some sort of program is needed to show, for example, variation over time, then the spreadsheet should be the first choice.

3 Packages vs Spreadsheets

The Association for Educational Communications and Technology (AECT) has a useful website, and, in particular, an interesting page [201] entitled *Spreadsheets as cognitive tools*. On it, we read the following quotation from the doctoral dissertation of Leon Argyla [109]:

Spreadsheets are powerful problem-solving tools. However, the difficulty in using spreadsheets for problem solving depends on the amount of abstractness and information processing the problem contains (Leon-Argyla, 1988).

At the same time as spreadsheets, a number of computer packages have been developed that parallel business and educational applications. In the arenas of statistics and geometry, sites such as CTI$^2$ [202] and CIGS$^3$ [203] show that specially designed packages are preferred to the computational and graphical features of spreadsheets.

However, on the benefit side, we would suggest that:

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2 Computers in Teaching Initiative

3 Corner for Interactive Geometry Software

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1. Building spreadsheets requires abstract reasoning by the learner.

2. Spreadsheets are rule-using tools that require that users become rule-makers (Vockell and van Deusen [192]).

3. Spreadsheets promote more open-ended investigations, problem-oriented activities, and active learning by students (Beare [31]).

Feicht [67] describes the use of Excel to investigate relationships between matrices and geometric transformations, and in doing so, opens up a very rich field for further investigation. Today’s mathematical syllabus has little time for geometry, but it may see a resurgence if it can be shown that electronic versions of theorems have the power to excite the imagination and enthusiasm of learners—see the mathematics site of Bogomolny [204], which is testimony to the power of animating concepts in mathematics and statistics. For an example hosted in Microsoft Excel, consider the work of Staples [175]. We have seen that when it comes to programming, spreadsheets have desirable features; it may be that in the future, the animation of concepts is an area in which spreadsheets will have an increasingly influential role over computer packages.

4 Mathematics Education

For applications of spreadsheets in mathematics education, especially that of mathematics educators, Abramovich, along with co-workers, is the most prolific author ([1]–[22], [110]). Indeed, Abramovich and co-workers are leading the field in this area. The website [1] for Abramovich’s course “Using Spreadsheets in Teaching School Mathematics” contains a number of useful links, and is well-worth investigating. Abramovich and Brantlinger [11] examine the suitability of spreadsheets for mathematics teacher education. They note that the typical classroom is not well-equipped when it comes to a range of mathematical software.

The appearance of computers as alternative instructional tools has created serious problems in undergraduate pedagogy. This is particularly the case for mathematics teacher education, because technology as a mathematical/pedagogical tool is not always used appropriately in schools. Indeed, some teachers who attempt to incorporate technology into the curriculum limit its use to routine computations only due to a lack of experience with this technology. The mathematics education community views this problem as a great challenge to educational reform [11].

In [10], the same authors give examples of the use of recursion in a spreadsheet, the visualization of the limit of a sequence, and cobweb diagrams. The paper concludes that, given the wide availability of spreadsheets, a toolkit of computer resources (such as a scientific calculator, a graphing package and a database program) can be just one program: a modern spreadsheet. Abramovich and Norton [15] extend this work to investigation of chaos, and associated basic concepts of convergence, divergence, cycling and period doubling. They further argue that for mathematics education majors, the spreadsheet is the ideal vehicle for the illustration of mathematical fundamentals such as these. Similar topics are covered in another paper by Abramovich et al [13].

4.1 Collaborative Constructivist Learning

D’Souza and Wood [200] investigate the benefits of spreadsheets in a collaborative learning environment at secondary school level, with the application being elementary financial mathematics. The authors state that:

Spreadsheets have enormous potential for assisting in the learning of algebraic concepts. They can be of great benefit at all levels. Spreadsheets enable students to concentrate on thinking about the subject matter at hand rather than on the software. There are many mathematical applications of spreadsheets as noted by Beare [32].
It is worth repeating the relevant citation from Beare [32]:

Spreadsheets... have a number of very significant benefits many of which should now be apparent. Firstly they facilitate a variety of learning styles which can be characterised by the terms: open-ended, problem-oriented, constructivist, investigative, discovery oriented, active and student-centred. In addition they offer the following additional benefits: they are interactive; they give immediate feedback to changing data or formulae; they enable data, formulae and graphical output to be available on the screen at once; they give students a large measure of control and ownership over their learning; and they can solve complex problems and handle large amounts of data without any need for programming.

Herrington and Standen [84] consider that many multimedia educational packages tend to present material in an instructivist manner, thus placing the learner in a passive role. They would prefer to see learning posed in an authentic setting to provide a constructivist learning environment, and it is the spreadsheet that provides just such an environment4.

In a 1997 conference presentation, Sher [165] asserts that:

The spreadsheet is the ideal environment for software that follows the Harvard approach.

In a nutshell, the so-called Harvard approach [for mathematics teaching] is that every topic should be presented geometrically, numerically, and algebraically. Such an approach is exemplified in the work of Hughes-Hallett [89] and many others. Similarly, Friedlander [72] suggests that:

Spreadsheets build an ideal bridge between arithmetic and algebra and allow the student free movement between the two worlds. Students look for patterns, construct algebraic expressions, generalize concepts, justify conjectures, and establish the equivalence of two models as intrinsic and meaningful needs rather than as arbitrary requirements posed by the teacher.

Thus, in mathematics education, we are seeing the emergence of the spreadsheet as a key component of the constructivist teaching repertoire. There is no doubt that this role will grow in importance, as students become more and more competent in the mechanics of using spreadsheets.

We close this section with some comments from a relatively modern work, that of Ainley et al [24]:

Over recent years, we have explored the conjecture that particular pedagogical settings that exploit immediate and continuous access to computers can change the way in which knowledge about graphs is constructed.

...the spreadsheet provides the facilities that allowed Clara and Colin to construct these meanings for trend ...

...Through activities with spreadsheets in this pedagogic setting, children expressed new meanings in which scattergraphs became increasingly powerful tools for analysing ongoing experiments.

4.2 K-12 mathematics

Lewis [114] notes that:

Spreadsheet assignments offer concrete ways to explore abstract concepts in mathematics and other subjects.

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4 We are not implying here that spreadsheets are the silver bullet that will transform traditional classrooms into constructivist ones. Indeed, it is quite possible to imagine students being taught to program spreadsheets in teacher-centred ways.
Lewis [115] has also produced a spreadsheet resource book for mathematics teachers of grades K-8. Nowhere is the exploration of an abstract concept more fully researched than in the study of Sutherland and Rojano [184] in which the potential of the spreadsheet to enable students to form a correct understanding of algebraic concepts such as pronumerals is investigated. This frequently cited work, in a sense, sets the benchmark for educational studies of spreadsheet potential. Sutherland [187] has much more recently used the spreadsheet environment to allow secondary school students in the UK to develop basic concepts of algebraic dependency. Since the students have trouble with the abstract nature of algebra, the spreadsheet is used to develop relationships with point-and-click. Note the following remarks from Sutherland’s paper [187]:

The more traditional approach to teaching algebra often involved imposing an algebraic method with an over-emphasis on the manipulation of symbols and with no acknowledgement of the value of the pupils’ own approaches. This way of teaching algebra was only successful for the minority of pupils.

Our experimental work has shown that most pupils do not spontaneously engage in the algebraic processes of expressing generality, acknowledging and manipulating the unknown, focusing on structure, using an analytic algebraic method.

One way to help pupils move from a non-algebraic to an algebraic approach can be through work with spreadsheets. . . . Pupils use the mouse to support the expression of general relationships and to move from thinking with situation-based to more abstract algebraic objects. . . . Mouse pointing becomes a way of supporting pupils to express general relationships, which are then represented automatically in spreadsheet code. Pupils become aware of this spreadsheet code without explicit instruction and interact with it when they need to modify their constructions. They begin to use the spreadsheet code in their talk when communicating with their partner and can write it down when communicating with others. In this way the algebra-like spreadsheet code is learned effortlessly without explicit teaching. Pupils use the spreadsheet specific calculations to help in the construction of general rules and often verify their general rule with reference to specific numbers. In this way links between symbols and general numbers are established.

Similar comments are made by Abramovich [12]. Referring to a spreadsheet model to support an inductive proof, it is stated that the model:

. . . allows for the visualization of an inductive proof of combinatorial identity, and it cognitively supports a transition from computing to a formal language of mathematics.

Ruthven & Hennessy [162] give an account of the use of various forms of computer technology and software to support mathematics learning in seven English secondary schools, in the year 2000. In this rather lengthy paper, although there is much interesting material, we generally confine ourselves here to aspects relevant to spreadsheets. Some of the major points are:

1. Students are nowadays more familiar with basic IT applications, including spreadsheets, so that much less time is spent on trivial matters in Excel, and the class can spend most of its time on mathematical activities within the spreadsheet. This is in contrast to the graphics calculator, which is not as ubiquitous as the spreadsheet, and is typically only used in mathematics classes, whereas the spreadsheet could be used in IT skills courses, business, economics, etc.; it is a more generic tool.

2. Teacher and students have more control over spreadsheet models, e.g., sequences, than for other software.

3. In a comparison of classes of software used for mathematics instruction and modelling in the seven schools (courseware, graphware, Logo, spreadsheet), the spreadsheet was the only one in use at all the schools.
4. A teacher at one school had his year 11 students doing statistical text processing in Excel: some text from newspaper website was transferred into Excel, then analyzed statistically, in a short space of time.

In hindsight it seems obvious, but probably one of the most profound, clear benefits of using spreadsheets that emerges from this study is just that of saving time. The time gained can then be spent on investigating properties of the mathematical objects created in the spreadsheet environment: the so-called what-if scenarios. There is huge scope for investigation of dependence on parameters in almost any spreadsheet model of a mathematical process. This includes not only the “traditional” mathematics that we might classify under the headings of algebra, calculus, trigonometry, geometry etc., but also more modern and discrete areas such as string processing, language theory, automata theory, and of course, combinatorics and recurrence relations. Note the following comments from a teacher of years 7 and 8:

We’ve used spreadsheets in Year 7 and 8, to enable them to look at handling data, because they can quickly get tables and produce charts that are much better quality than those that they can produce themselves. I’ve got the bottom set in Year 7 and it can take them the whole lesson to draw a bar chart. So it’s particularly successful from that point of view because they don’t have to draw all the axes so much, and it doesn’t take them so long to develop the ideas because they’re not having to spend a whole lesson drawing something. They can draw twenty graphs in a lesson and actually see connections, rather than spend twenty minutes drawing the axes and then twenty minutes talking and then twenty minutes drawing all the graph.

It saves a lot of time as well with the Further Maths and the graphing that we did. It would have taken forever to actually plot all the points and see what happens when you transform certain shapes. Whereas it was done in a flash and they could see and they learnt an awful lot. So then they were ready and they’d accepted it because they’d seen it happening... Whereas it would have taken many lessons if we’d actually plotted all these graphs, they’d have just got bored by it. So that definitely helped, just kept the pace going. [162]

It needs to be added that, despite the growing number of studies that show positive results for the use of spreadsheets, curriculum goals such as those produced by State and National Education Departments place less emphasis on the role of spreadsheets than on that of calculators as a tool for social constructivism in mathematics. There is little doubt that the main reason for this emphasis on calculators is based on educational administrators being constrained by the number of computers that are available in any given classroom. As a referee of this paper has noted: “it is not that graphing calculators are a superior tool it is rather that graphing calculators are a more accessible tool.” However, curriculum administrators are beginning to acknowledge the enormous potential of the modern spreadsheet program to enhance learning opportunities.

For a review of the K-12 uptake of spreadsheets, the ERIC digests by Ozgun-Koca [145] for the years 2000 and 2001 make a good starting point, as they contain numerous links to web-based resources. The book [101] is a good source of spreadsheet examples for grades 3 to 8. See also the work of Dugdale [59].

4.3 Number Theory

In spite of number theorist G.H. Hardy’s claim [82] that “I have never done anything useful”, the utility of number theory nowadays is undeniable, with prime applications being mathematical cryptography and cryptology. Modern CS/IT graduates need to know at least the basics of cryptographic computer security systems, and the use of spreadsheets in particular for the illustration of number-theoretic concepts appears to be on the rise. Just a few examples are the work of Abramovich and co-workers, cited earlier, and that of Sugden [182]. Sugden uses Excel for illustration of Euler’s \( \varphi (n) \) (totient) function, \( \tau (n) \) (number of divisors), the Möbius function \( \mu (n) \), as well as the basic
operations of modular arithmetic, including modular inverse, modular exponentiation, leading to a 32-bit implementation of the RSA public key cryptosystem in Excel.

Abramovich [7] also considers the use of Excel for some basic investigations into number theory, and gives an implementation of the Sieve of Eratosthenes using a few very simple formulas. He goes on to consider gaps between primes, primes in arithmetic progression, even showing how to tabulate and graph the function $\pi(x)$ (number of primes less than $x$), all in Excel. His formula for this latter task is very long but consists of a sequence of very similar sub-formulas, and one suspects that this could be simplified somewhat, given the modern facilities of, for example, Microsoft Excel XP. In his article, Abramovich quotes earlier authors MacKinnon [119]:

\[ \ldots \text{the best use of computers in mathematics education is to run professional software written for real purposes.} \]

and Steward [176] (op cit, p20). In a later paper, Abramovich and Brantlinger [11] present further topics in elementary number theory within the spreadsheet environment. These include Pythagorean triads, Euclidean algorithm, Bride’s Chair, and the representation of integers as the sum of perfect powers.

### 4.4 Combinatorics

Neuwirth [135] showed that:

\[ \ldots \text{spreadsheets as a model for mathematical relations can help gaining insight into recursive relations for combinatorial formulas.} \]

This paper and Neuwirth [136] use the very fundamental features of a spreadsheet to illustrate the structure of elementary recursive combinatorial identities, introducing an arrow diagram notation to demonstrate the concept:

\[ \text{We see immediately that in this graphical notation the formula is the same for any case in a very intuitive sense.} \]

Neuwirth aims to avoid algebra as much as possible as the students have trouble with this and with the closely related concept of mathematical induction. Sugden [179] also uses Excel to assist with investigations into mathematical induction and recursive implementations of arithmetic and geometric sequences (directly applicable to simple and compound interest respectively). The concept of a recursive relation is one that lies at the heart of Microsoft Excel’s Fill Handle implementation, which takes a range of cells in which formulae apply and spreads them over a range in a way that replicates the recursive process. We believe that an investigation into student understanding of recursion can be greatly enhanced by implementing such a process on the spreadsheet. For example, Sugden [179] uses this technique for his discrete and business mathematics classes to implement a full superannuation model, complete with rollover and schedule of net worth, in Microsoft Excel, with just one addition and one multiplication—no exponentials or intrinsic functions. Given such an environment, the traditional conceptual difficulty of recursion is replaced by the visual simplicity of a single-step formula (one recursive step) and Excel’s fill-down. Hvorecký and Trencansky [94] use cell insertion in Excel, under VBA/macro control, to investigate graphs of functions of one and two variables, and to the generation of fractals. As further cells are inserted (their requirement for extra cells grows exponentially), graphs and fractals of higher precision are obtained.
4.5 Numerical Analysis

There is a healthy literature on the uses of spreadsheets in numerical analysis (NA). A topical example would be Lawson and Tabor [108], in which a model arithmetic and geometric growth is presented using a spreadsheet. Their application is to model the spread of Variant Creutzfeldt-Jakob disease (mad-cow disease, or vCJD). In all references located, the positive aspects of spreadsheet use recur. For example, Soper and Lee [178] conclude:

> The versatility of computer spreadsheets makes them a very suitable means of accurately undertaking numerical calculations. The formulae required are easily entered into the spreadsheet, and the approach has the further advantage that the users can see what is being calculated, so enhancing their understanding of the method. With the availability of graphical facilities, a plot of the value of a variable at successive iterations can easily be displayed. A further benefit of the approach is that students become proficient in the use of a very popular business computer package.

Another two examples of using spreadsheets for numerical analysis are from Australia: Smith [174] describes an entire NA course at Monash University, Australia (1989) taught entirely using the spreadsheet program VP-PLANNER, and McLaren’s book [126] has been used to support his NA course at LaTrobe University.

Sequences, a key element of numerical modelling, are a natural for a program such as Microsoft Excel. There is no doubt that the modelling of sequences is one of the great strengths of the modern spreadsheet program; in fact, even the very early versions such as VisiCalc were almost as good. Sequences can be defined by just listing the terms, by a direct formula, or by a recurrence. For the most basic of sequences (arithmetic and geometric), the most natural definition is the recursive one: start anywhere and either keep adding a constant or keep multiplying by a constant. From the point of view of student understanding, such a prescription is almost too easy to implement in a modern spreadsheet such as Microsoft Excel.

For the 21st century, IT-literate student, it must seem like a total a waste of time that the teacher has to write a recurrence relation to express such a simple operation as fill-down! Students familiar with spreadsheets already know all about this. The challenge for the teacher is, of course, to relate the mathematical formalism of a recurrence relation to the readily comprehended spreadsheet notion of fill-down. Student reaction to the translation of mathematical formalisms such as the algebraic expression of a recurrence relation such as that of Equation 1 into the spreadsheet environment are discussed briefly by Sugden [179]. Equation 1 represents a superannuation model with $100,000 rollover (initial value), 1% interest per month, and $500 contribution per month. It is interesting that many students have difficulty even understanding Equation 1 (let alone solving it), yet have very little trouble implementing the corresponding model in Microsoft Excel. Clearly, there is ample scope for future research here.

$$a_n = \begin{cases} 1.01a_{n-1} + 500 & \text{if } n > 0 \\ 100,000 & \text{if } n = 0 \end{cases} \quad (1)$$

It is clear that immediate application of such recursion is to be found in the area of elementary mathematics of finance. Coupled recurrence relations (difference equations) relating to population dynamics, for example, are also easily handled. Since many sequences are most naturally defined recursively (arithmetic, geometric, Fibonacci etc.), the spreadsheet offers a very rich environment for investigation of sequences, in most instances, with no coding required. Further discussion may be found in [180].

4.6 Mathematics not otherwise cited

Due to limitations of time, it was not possible to obtain a copy of every publication considered relevant to the application of spreadsheets to mathematics teaching and learning. However, in the
interests of providing as complete a bibliography as possible in the present survey paper, the following additional works are cited: [30], [41], [58], [60], [61], [62], [63], [85], [86], [93], [96], [97], [117], [118], [120], [122], [127], [125], [132], [138], [139], [142], [143], [148], [151], [152], [158], [159], [161], [164], [172], [173], [185], [190], [191], [195], [197] and [199].

5 Statistics

Does the spreadsheet have advantages that make it a more acceptable tool for teaching and learning than the use of statistical packages? In this respect, Hunt [91] refers to teacher belief as follows:

Many teachers now believe that a spreadsheet provides a better educational environment in which to teach statistics at an elementary level. The spreadsheet can be made much more transparent to the student, allowing them to look inside the black box.

Overall, however, the jury is still out on this question—at least at the tertiary level. Although Neuwirth [137], for example, has demonstrated that spreadsheets may be employed to assist statistical learning beyond just that of using intrinsic functions (he writes about visualizing correlation), there is a good deal of negativity in the literature when it comes to statistical computations. For example, writing about versions of Excel 5.0 and earlier, Nash and Quon [134] conclude that:

Spreadsheet vendors must be encouraged to do better. Closer attention to statistical issues would result in tools better suited to data exploration and analysis, and cleaner software design would avoid some obvious sources of error.

They would like to see options for:

1. Histograms, possibly with unequal class intervals.
2. Stem and leaf diagrams.
3. Boxplots, especially with multiple boxplots on the same scale.
4. Quality-control charts.
5. $p - p$ and $q - q$ plots for distributions, especially the Gaussian distribution.

If the requirement, even at tertiary level, is to cover only elementary concepts, others find that the spreadsheet serves admirably. For example, in teaching an engineering subject, Hall [81] concludes that:

This approach to teaching mathematics, with an intimate mix of mathematical theory, numerical examples and graphical representations, together with the use of modern computing aids, seems appropriate for engineers.

Warner and Meehan [196] express similar support, stating that:

Individual instructors will need to weigh the costs and benefits of using a spreadsheet program versus specialized statistical package, but we believe a spreadsheet, such as Excel, will prove more attractive in many situations.

Hunt [92] reasserts the belief of teachers, concluding that although Microsoft Excel has a great deal to offer at the elementary level, it is of limited use to serious students of the subject. He points to features such as NORMDIST(1.96) which returns the tail probability of 0.975 and commends the function NORMSINV(RAND()) which can be used for simulating a standard normal random deviate.
Features such as these really do make the spreadsheet a simple-to-use tool for the constructivist learning environment. Hunt also identifies two types of computer-assisted learning activities that are well-supported by spreadsheets. There is the ‘do-it-yourself’ type of activity favoured by Callender and Jackson [49], and the ‘Blue Peter\(^5\)’ activities where students experiment with pre-set spreadsheets such as the DISCUS materials of Hunt and Tyrrell [90].

Callender [48] demonstrates how the spreadsheet, with simple-to-write macros, can be used to demonstrate the normal distributions and the central limit theorem. He suggests that a benefit of using spreadsheets to introduce such key topics in statistics is:

> The normal distribution has been introduced without ‘definitions’ being needed and all the components of the Central Limit Theorem are demonstrated as a consequence of probability sampling.

However, a recurring complaint is that Microsoft Excel does not cater for exploratory data analysis. We would question the value in Excel’s catering for such a specific and small audience as that for whom exploratory data analysis is an issue. Features such as box and whisker plots can readily be constructed on XY-scatter charts, and once the framework for such a plot has been made, it can be quickly copied for use with different data. To make a histogram with uneven intervals, our advice would be ‘Don’t!’ and if you have data that needs to be summarized on a histogram, the Pivot Table facility provides a very speedy way of generating the required data table. It would appear that the true value of these educational oddities of exploratory data analysis have not been recognized by the wider business community, and hence not implemented by Microsoft programmers.

More serious is the common complaint that Excel statistical functions are unreliable [123], [124], [104]. Indeed the present authors would hesitate to recommend any version of Microsoft Excel for professional statistics work, although the second author has done much consulting work based (stochastic modelling) in Excel [181] and the package performed very well. But, from a pedagogical standpoint, it is here argued that a collection of totally robust algorithms for computing statistical functions and distributions is not the principal requirement for students when learning basic statistical techniques and functions. Provided the implemented functions are reasonably accurate for normal ranges of the parameters, it is far more important that the student is able to understand the operation of such functions and so see the many connections, patterns and properties; these are amply illustrated in the spreadsheet environment. However, a revamp of the statistical functions appears to be on Microsoft’s wish list for Excel 2003 [193].

In this context, please forgive us for slipping in a positive comment about the MODE function; one that is taught in very elementary statistics courses, never again to rear its head. In Excel, however, we find that the MODE function is a powerful tool for locating duplicates in a list, particularly when combined with conditional formatting (e.g., if the value of this cell is =MODE(range) then format the cell to have a blue colour). Some further useful information concerning the use of conditional formatting with Excel’s MODE function was pointed out by an anonymous reviewer, and is included here, with permission.

> It appears that in the case of poly-modal array of numbers, the MODE function recognizes the first number that appears most in this array. For example, using conditional formatting jointly with this function, in the array \{2, 1, 3, 2, 1, 5, 3, 4, 4\} twos will be highlighted only, whereas in the array \{1, 2, 3, 2, 1, 5, 5, 3, 4, 4\} ones will be highlighted only.

It would be inappropriate to complete this section without mentioning the Association of Statistics Specialists Using Microsoft Excel (ASSUME) [205] whose main interest appears to be the use of Excel in higher education. Queries to the ASSUME list will almost always receive an answer from someone who knows.

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\(^5\)This name refers to the English children’s TV program ‘Blue Peter’ catchphrase – “Here is one that I prepared earlier".
6 Physical Sciences

Spreadsheets have long been used both for teaching mathematical principles of physical science and analysis of empirical results; for example [27], [31], [34], [35], [39], [40], [42], [43], [44], [46], [47], [50], [51], [52], [53], [56], [57], [65], [66], [68], [69], [75], [77], [79], [80], [99], [100], [102], [103], [105], [108], [111], [128], [129], [130], [131], [133], [140], [141], [144], [149], [153], [163], [169], [188], and [198]. However, in his 1992 paper, Smith [171] notes the comparative rarity of using the spreadsheet as a mathematical teaching tool. He goes on to mention some uses of spreadsheet in teaching physics, and cites the texts of Misner & Cooney [130] and Crow [52]. In his closing remarks, Smith predicts three positive outcomes for students:

1. Reversal of the declining interest in mathematics.
2. Improvement of technological literacy and enhancement of career preparation.
3. Revitalization of mathematical skills through problem solving.

Some textbooks, not mentioned elsewhere, and which employ spreadsheet-based models, algorithms and solutions are [25], [26], [27], [33], [37], [38], [54], [55], [83], [106], [107], [150], [155], [170], [183], and [186].

An interesting modern paper is that of Lim [116], which describes how spreadsheets are used to support the teaching of quantum chemistry. It is reported that students with weak mathematical backgrounds have profited from this approach.

This author prefers the use of spreadsheets for weaker students for the following reasons. The symbolic mathematical packages depend on the use of a symbolic, quasi-programming language, which can present an additional learning obstacle for many students....

Furthermore, the access to symbolic mathematical packages is usually more limited than that of spreadsheets, which are widely available in home, business and community settings. The “worldware” ...also called “application-software” ... nature of spreadsheets means that students will have greater opportunities to use and become familiar with spreadsheets than with (eg) symbolic mathematical packages, leading to greater utility and expertise. Software that isn’t designed for instruction can still be good for learning....

A “straw poll” of physical chemistry faculty suggests that significantly more faculty use spreadsheets in teaching and learning activities than symbolic mathematical packages [116].

7 Computer Science

Shinners-Kennedy [166] reports on the use of early work in which spreadsheets provide a host environment for teaching assembly language programming. Even as early as 1986, he notes:

It is surprising the range of applications that have been coaxed into this format.

In another paper [167], the same author describes computer programming and data structures and algorithms courses and refers to the well-known obstacles to comprehension of run-time behaviour of programs for "weaker students". The response of several educators to this problem is the creation of run-time visualization environments. Similar considerations are expressed by Rautama et al [160]. Once again, the authors state that many students have trouble understanding basic algorithms, and it is a significant challenge to inspire them to investigate why and how a given algorithm works. Examples they cite are binary search, and edit distance for string editing. Rautama et al use Microsoft Excel as an algorithm animation environment, and also point out that such an approach leads to useful tools for algorithm research too. Shinners-Kennedy [167] recommends the use of spreadsheets for construction of such visualization systems, and goes on to describe the benefits of the Microsoft Excel Object Library for such endeavours:
The spreadsheet concept is deceptively simple.
In essence the spreadsheet system is a toolkit (emphasis ours) for exposing and explaining the principal concepts associated with the object-oriented methodology.

Fone [70] demonstrates the value of using Excel to model neural networks. In an end-of-course survey, Fone found that:

Unlike previous groups, there were no comments suggesting the need for better models. However, five comments typified by the statement, “I found the Excel example particularly helpful, not only in helping me understand the networks but with aspects of other modules” were of particular interest.

Other applications of spreadsheets in computer science are described in [95] (programming), [112] (arithmetic unit simulation), [113] (automata), [168] (object-oriented programming), [64] (animation).

8 Economics and Operations Research

Thiriez [189], in an invited review of the role of spreadsheets in operations research teaching begins by pointing out that there is:

...no point in demonstrating the efficiency of the spreadsheet as a tool for teaching basic modeling and programming.

Thiriez also notes that there are less well-known features of spreadsheets that turn out to be powerful, so long as you know how to use such features. He goes on to give examples in decision theory, linear programming and simulation:

The spreadsheet is the best tool for teaching deterministic simulation.

Suggestions are also made that users should view the Solver capabilities with caution. However, his key point is that:

A major asset of spreadsheets resides in the speed with which a model may be designed. Recently, I had the opportunity to build a simulation model for the automatic handling of luggage at the Paris Roissy airport. When the model (totally developed in Excel) was finished, I learned that all other proposals for the tender offer of this application, all of them based on the use of professional simulation software, had all been at least five times as expensive as our proposal.

Many other examples of the use of Excel’s Solver are to be found on the WWW; for example, see [177].

Jones and Judge [98] support the general views of Thiriez in a paper that gives examples from micro-economics and macro-economics, illustrating how a spreadsheet can be used to assist in the development of a student’s understanding of dynamic models and their properties. Adams and Kroch [23] point to the value of the graphical component of spreadsheets:

The linkage between algebraic, numerical and graphical presentations in spreadsheet programs on the personal computer is a signal advantage for the teaching of macroeconomics ... to illustrate the principal elements of macrotheory.

Goddard et al [78] note the way in which spreadsheet usage supports a shift from active to passive learning by enabling students to explore what happens to income-expenditure economy models (the IS-LM model) as equilibrium is disturbed.

Barreto and others use Excel for the teaching of Economics and Econometrics at Wabash College [29].
9 Some spreadsheet esoterica

9.1 Conditional formatting

An interesting feature, introduced in Excel 97 is that of *conditional formatting*. It may be viewed as a generalization of the common accounting practice of colouring amounts of money red or black, depending on their natures as debits or credits (negative or positive). Microsoft Excel is able to automatically format (colour, border, font etc.) a cell based on its current value. A simple example of the use of this feature is solving \( f(x) = 0 \) without algebra, but by just observing change of sign (change of colour). Those interested in the use of this facility will find downloadable models on the *Spreadsheets in Education* (SIE) eJournal site [28]. The literature on the use of this feature for educational applications within the spreadsheet environment is scant indeed, although applications abound. One reference to its use is that of Sugden [179], who uses it for solving \( f(x) = 0 \) without algebra, and illustrating the solution of simultaneous linear congruences, among many others.

9.2 Names

One of the most beneficial features of the modern spreadsheet is the facility to define a *Name*. Such a feature allows the spreadsheet user to refer to a cell or collection of cells by a single identifier; this is similar to the use of variable names in programming languages rather than hard addresses (relative or absolute). Many benefits accrue with the use of names; some of these being:

- The troublesome, but sometimes necessary, *absolute references* are handled automatically.
- Models may be expressed in notation very close to that of standard algebra. Instead of using an obscure formula with hard cell references such as \( = A2*$E$3 \), one may write something like \( = \text{GrossPay} \times \text{TaxRate} \).
- Large areas of the worksheet such as tables or lists may be easily selected by just going to the Name box (Microsoft Excel).
- Models become at least partially self documenting and tend to be easier to debug.

Despite the fact that problems with relative and absolute addressing simply vanish, and that formulas may be expressed in terms very close to an algebraic model when one uses names, it is rather surprising how rarely these are used in published examples.

9.3 Auditing and debugging

In this paper, we have trumpeted the benefits of spreadsheets for educational purposes. What is the downside? There exist quite a number of papers which highlight negative aspects of spreadsheets, and in the interests of a more balanced presentation, it seems only fair that our survey should include some of these papers too. The main objection appears to be *debugging* (to use a computer programmer’s term), or *auditing* (to use an accounting term). When applied to spreadsheets, these two terms refer to much the same thing: the problem of ensuring that a given model is correct.

The difficulties of auditing spreadsheets are well-known, and the level of complexity seems to be comparable to that of debugging a moderate-sized computer program; this problem is well-known to be hard [73], [74], [146], [147], [156].

To put our negative remarks about spreadsheet auditing in perspective, we note that modern spreadsheets such as Microsoft Excel have some quite useful auditing tools; for example, even without invoking the *auditing toolbar*, just a double-click in a cell will highlight immediate antecedents, that is, cells which directly affect the value of the current cell. Secondly, spreadsheet models developed for educational purposes tend to be founded on a sound mathematical model, and if wrong results are produced, or a limitation of the model is reached, then students can be asked to investigate why such
a happening has occurred. In other words, in an interactive learning environment, it is often possible to turn an apparently negative outcome to a positive one. For an example of this, see Sugden [179].

10 Conclusion

In this brief outline of spreadsheets in education, one of our principal aims has been to provide arguments and motivation for further research in this area. There is no longer a need to question the potential for spreadsheets to enhance the quality and experience of learning that is offered to students. Traditional barriers (particularly the lack of facilities to use spreadsheets in assessment contexts) need to be removed, either by ensuring that access to computers is improved or by changing assessment methods. Further expansion is needed of the types of topics that can be effectively covered by spreadsheet examples; for example, one of the authors has recently completed a spreadsheet to enable the investigation of cellular automata as described by Gardner [76].

Hence the electronic journal *Spreadsheets in Education*. The goals of the journal [28] are:

1. To create a forum in which ideas on the use of spreadsheets can be exposed and explored.
2. To provide an avenue for scholarly research into the use of spreadsheets at all levels of education to be reported to a receptive, practising audience.
3. To enable teachers to take on board a technology which can rightly claim to be one of the founding fathers of the personal computer and which is a tool for life to which all students should be exposed.

The electronic medium was chosen over print so as to provide a facility to discuss spreadsheets in education and at the same time giving access to the spreadsheets under discussion. In the view of the authors, to do otherwise would be inappropriate in this age of technology.

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[205] URL: http://www.jiscmail.ac.uk.
Extensive research supports the use of spreadsheets in mathematics classrooms to empower student-centered learning. J. C. Russell, in Spreadsheet Activities in Middle School Mathematics (NCTM, 1992), observed that a blank spreadsheet can be seen as a “student of the student,” capable of being taught, as directed by the student. Useful quotes and references are cited in Using Spreadsheets in Mathematics Education, ERIC Clearinghouse for Math, Science, and Environmental Education (Columbus, Ohio, 2000 - https://www.ericdigests.org/2003-1/math.htm). The NCTM's 2000 Principles and Standards recommend that "students in grades 3-5 should also become familiar with technological tools such as dynamic geometry software and spreadsheets. Education, the editors have compiled this overview of the use of spreadsheets in education. The aim is to provide a comprehensive bibliography and springboard from which others may develop. Spreadsheets in Education—the First 25 Years. Essentially the same points are made in a more recent paper by Steward [176]: I would suggest that when both are possible, students find it easier and quicker to use a.