Effective theory of the fractional quantum Hall effect

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1. Introduction

The quantum Hall effect is a fascinating quantum phenomenon observed in two-dimensional electron systems under a strong magnetic field, marked with high precision quantization of the Hall conductance to integer and some fractional multiples of e^2/h. The integer quantum Hall effect is most naturally explained in terms of an interplay of localization and Landau quantization (i.e., quantization of electron's cyclotron motion in a magnetic field). The fractional quantum Hall effect [1] (FQHE) indicates formation of novel quantum fluids due to electron-electron interactions. The key feature of the quantum Hall states, common to both integer and fractional cases, is that they are incompressible fluid states. The early study based on Laughlin's wave functions [2] revealed all such basic characteristics of the FQH states and gave impetus to new pictures [3] of the FQHE in terms of electron-flux composites, composite bosons and composites fermions.

The Chern-Simons (CS) theories [4], both bosonic and fermionic, are the field-theoretic frameworks that realize the composite-boson and composite-fermion descriptions of the FQHE and have been successful in describing various features of the FQH states. They, however, have some subtle limitations as well [5]. In particular, when applied to bilayer systems, they differ significantly in collective-excitation spectrum from the magneto-roton theory of Girvin, MacDonald and Platzman [6], based on the single-mode approximation (SMA).

In this talk we wish to report on an approach [7] to effective theories of the FQHE, that relies on the incompressible nature of the quantum Hall states without referring to the composite-boson or composite-fermion picture. The basic tools used are projection to the Landau levels, functional bosonization [8] and the SMA, which are combined to construct, via the electromagnetic response of the incompressible states, a long-wavelength effective theory of the FQHE. For single-layer systems this effective theory properly reproduces the results consistent with the CS theories. Our approach thus has logic independent of but complementary to the standard CS theories.

The real merit of our approach becomes prominent for bilayer systems. We shall examine the electromagnetic characteristics of bilayer systems within the SMA theory and derive an effective theory that properly incorporates the SMA spectrum of collective excitations. We shall thereby clarify the relation between the SMA theory and the CS theories. In particular, we point out that subtleties and limitations of the latter are traced back to the fact that the CS approach, making no explicit use of Landau-level projection, fails to distinguish between the inter-Landau-level and intra-Landau-level modes.

2. Electromagnetic response of Hall electrons

Consider electrons in a plane with a strong perpendicular magnetic field $B > 0$, and study how they respond to weak external potentials $A_\mu(x) = (A_0, \mathbf{A})$. The eigenstates of a freely orbiting electron are Landau levels of energy $\omega_{\rm c}(n + \frac{1}{2})$ with $\omega_c \equiv eB/m^*$ and $n = 0, 1, \cdots$. This level structure is modified in the presence of $A_\mu(x)$, and the effect of level mixing it causes is calculated by diagonalizing the Hamiltonian H (with A_μ kept) with respect to the true levels $\{n\}$. This procedure is called projection.

Suppose now that an incompressible many-body state of uniform density $\rho_{\rm av}$ is formed via the Coulomb interaction within the lowest Landau level. From the projected Hamiltonian $\bar{H}$ one can then read off its response due to electromagnetic inter-Landau-level mixing, i.e., due to the cyclotron modes:

$$S^{\rm cycl} = \int dt d^2\mathbf{x}\, \rho_{\rm av} \left[-eA_0 + \frac{e\ell^2}{2} A_\mu D \epsilon^{\mu\nu\rho} \partial_\nu A_\rho - \frac{\ell^2}{2\omega_c} A_{k0}\, D A_{k0} + \cdots \right], \quad (1)$$

where $\ell \equiv 1/\sqrt{eB}$; $D = \omega_c^2/(\omega_c^2 + \partial_t^2)$, $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\epsilon^{012} = 1$.

The electromagnetic interaction in $\bar{H}$ also gives rise to intra-Landau-level transitions. For single-layer systems, however, the intra-Landau-level excitations are only dipole-inactive [6] (i.e., the response vanishes faster than $\mathbf{k}^2$ for $\mathbf{k} \to 0$) as a result of the f-sum rule or Kohn's theorem. This implies that quantum Hall (QH) states in single-layer systems show universal $O(\mathbf{k})$ and $O(\mathbf{k}^2)$ long-wavelength characteristics governed by the cyclotron mode alone.

3. Bosonization and effective theory

Once an electromagnetic response such as $S^{\rm cycl}$ is known it is possible to reconstruct it through the quantum fluctuations of a boson field, a procedure known as functional bosonization [8]. In the present case the cyclotron-mode contribution (1) is reconstructed from a bosonic theory of a 3-vector field $b_\mu = (b_0, b_1, b_2)$, with the Lagrangian

$$L_{\rm eff}[b] = -eA_\mu \epsilon^{\mu\nu\rho}\partial_\nu b_\rho - \frac{1}{\ell^2}\, b_0 + \frac{\pi}{\nu}\, b_\mu \epsilon^{\mu\nu\lambda} \partial_\nu b_\lambda + \frac{\pi}{\nu\,\omega}\, (b_{k0})^2 + \cdots, \quad (2)$$

where the filling factor $\nu = 2\pi\ell^2 \rho_{\rm av}$ and $b_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$.

This $L_{\rm eff}[b]$ almost precisely agrees with the dual-field Lagrangian of Lee and Zhang (LZ) [9], derived within the Chern-Simons theory and describing the low-energy features of the FQHE. We have thus arrived at the LZ dual Lagrangian without invoking the composite-boson picture.

4. Bilayer quantum Hall systems

The situation changes drastically for bilayer systems, where both in-phase and out-of-phase collective excitations arise over the two layers within the lowest Landau level. In-phase excitations generally remain dipole-inactive. Out-of-phase collective excitations, in contrast, become dipole active [5]. In particular, for typical bilayer QH states, as described by Halperin's (m, m, n) wave functions [10], there arises a dipole-active out-of-phase collective mode, with the spectral weight or the static structure factor

$$\bar{s}^-(\mathbf{k}) \equiv \frac{1}{2N_e} \langle G|\bar{\rho}^-_{-\mathbf{k}}\, \bar{\rho}^-_{\mathbf{k}}\, |G\rangle = c^-\, \frac{1}{2}\, \mathbf{k}^2 + O(|\mathbf{k}|^4) \quad (3)$$

with $c^- = n/(m-n)$; $\bar{\rho}^- = \bar{\rho}^{(1)} - \bar{\rho}^{(2)}$ stands for the density difference of the two layers. Experimentally bilayer QH states identified with the (3,3,1) state with $\nu = 1/2$ and the (1,1,1) state with $\nu = 1$ have been observed [11].

One can resort to the projected single-mode approximation (SMA) [6] [which proved reliable for QH systems as well as for liquid He] to calculate the electromagnetic response of bilayer systems due to intra-Landau-level processes. One obtains a dipole and related out-of-phase response of the form [7]

$$\begin{aligned} S^-_{\rm eff} = {} & \frac{\rho_{\rm av}}{2} \int d^3x\, A^-_{j0} \Big[\frac{2c^-\, \epsilon^-_{\rm coll}}{(\epsilon^-_{\rm coll})^2 - \omega^2} + \frac{\omega_c}{\omega_c^2 - \omega^2} \Big] A^-_{j0} \\ & - \frac{\rho_{\rm av}}{2} \int d^3x\, A^-_\mu \Big[\frac{2c^-\, (\epsilon^-_{\rm coll})^2}{(\epsilon^-_{\rm coll})^2 - \omega^2} + \frac{\omega_c^2}{\omega_c^2 - \omega^2} \Big] \epsilon^{\mu\nu\rho} \partial_\nu A^-_\rho + \cdots, \end{aligned} \quad (4)$$

where the cyclotron-mode contribution is also included; $A_\mu = A^{(1)}_\mu - A^{(2)}_\mu$ and $\omega = i\partial_t$.

From this response emerge the following observations:

(1) The presence of the dipole-active collective excitations, inherent to bilayer systems, modifies the leading $O(\mathbf{k})$ and $O(\mathbf{k}^2)$ characteristics of the bilayer systems substantially.

(2) The $O(\mathbf{k})$ response, now enhanced by a factor $(2c^{-}+1)$ which equals $(m+n)/(m-n)$ for the (m,m,n) states, precisely agrees with the result of the CS theories [12], whereas the $O(\mathbf{k}^2)$ response acquires the scale $\epsilon^{-}_{\rm coll} \sim e^2/\ell$ rather than ω_c. This explains why the CS theories correctly account for $O(\partial)$ features, such as the Hall conductance, vortex charges and long-range orders, while leaving the collective-excitation spectrum improperly on the scale of the Landau gap$\sim \omega_c$. In other words, the flux-attachment transformation, employed in the CS approach, properly introduces some crucial correlations among electrons, but unfortunately not all of them.

(3) One can reconstruct from such electromagnetic characteristics an effective gauge theory that consists of three vector fields representing one interlayer collective mode and two inter-Landau-level cyclotron modes. It properly incorporates the SMA spectrum of collective excitations, as well as the favorable transport properties of the standard CS theories.

A similar analysis has been extended [7] to bilayer systems in the presence of interlayer coherence and tunneling, where a variety of phenomena such as Josephson-like effects attract attention. The result again reveals some subtle deviations from the standard CS theories. All these subtleties are traced back to the fact that the CS approach, because of the lack of Landau-level projection, fails to distinguish between the inter-Landau-level (cyclotron) modes and the intra-Landau-level (collective) modes.

5. Interlayer Hall drag and collective excitations

The response (4) indicates that the interlayer collective excitations give rise to interlayer Hall drag, i.e., a current injected to the first layer induces a Hall voltage in the second layer. The effect is expected to be sizable for the $\nu = 1/2$ (3,3,1) bilayer state. No appreciable Hall drag, in contrast, is expected for the $\nu = 1$ (1,1,1) state from the SMA analysis (while a naive application of the CS theory suggests the contrary) [7]. It is desired to test these predictions by experiment.

6. Effective theory underlying the SMA

So far we have considered effective theories constructed from the electromagnetic response of QH systems. One might wonder if it is possible to derive them directly from the microscopic theory. The answer is yes [7]. The strategy is to note first that excitations in a quantum Hall system, both elementary and collective, are intimately connected with its Landau-level structure which is encoded in a huge symmetry, the $U(\infty)$ or W_∞ algebra. The single-mode approximation combined with the nonlinear realization of the W_∞ symmetry then allows one to derive the effective theories discussed above and, in addition, reveals some interesting similarity in structure (though not in scale) of the intra- and inter-Landau level excitations.

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X.-G. Wen Quantum Field Theory of Many-Body Systems, OUP (2004). An outline of basic material fol-lowed by an introduction to some advanced topics (topological order, the fractional quantum Hall effect, and spin liquids). A. M. M. Tsvelik Quantum Field Theory in Condensed Matter Physics, CUP (1995). A concise survey of applications of eld theory to condensed matter problems, especially in one dimension. We present three holographic constructions of fractional quantum Hall effect (FQHE) via string theory. The first model studies edge states in FQHE using supersymmetric domain walls in N=6 Chern-Simons theory. We show that D4-branes wrapped on CP1 or D8-branes wrapped on CP3 create edge states that shift the rank or the level of the gauge group, respectively.Â However, in the effective field theory description, the FQHE essentially occurs due to the presence of the Chern-Simons gauge field and its superpartners such as scalar fields and fermions do not contribute in any important way. Therefore, we can still capture the standard FQHE even in supersymmetric theories. Model II: holographic realization of level-rank duality and topological entanglement entropy. The fractional quantum Hall effect (FQHE) [3], i.e. a plateau in the Hall resistance, is observed in two-dimensional electron gases in high magnetic fields only when the mobile charged excitations have a gap in their excitation spectrum, so the system is incompressible (in the absence of disorder). Therefore the theory of the FQHE begins with the search for ground states of the interacting electron system which exhibit such a gap.