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Russell's Logic in *Tractatus logico-philosophicus*

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1. Wittgenstein in the wake of Russell's fame.

Ludwig Wittgenstein, as everybody remembers, was first known for his *Tractatus logico-philosophicus,* that is dependent on the formal logic of Bertrand Russell, and till the fifties nobody dared to approach that work, if he was not in some way acquainted with Russell and his logic system. As a consequence the name and fame of Wittgenstein partly enjoyed the renown and suffered the hostility that people had bestowed upon Russell.

The renown of Russell was mostly due to his achievements in the field of strictly formal logic and to his suggestion that formal logic is knowledge of the most intimate structure of the world we can know and think of. The hostility was a consequence of the destructions that such logic was thought to produce in the fields of ontology, moral and religion. Russell deeply experienced all along his life both the success coming from his renown and the harshness of his enemies.

The *Tractatus* of Wittgenstein was greeted by people that had mastered some knowledge of Russell's logic and had adopted the view that the thought of Wittgenstein was a development of the one of Russell and both had elaborated a sort of logical atomism or something like an introduction to logical positivism. Views of this sort greatly helped the diffusion of misleading interpretations of the *Tractatus* and of the very personality of Wittgenstein.

I can't forget how interesting were considered by many people in the fifties two bad papers by a philosopher named José

Ferrater Mora whose titles were "Wittgenstein o la destrucciòn"[1] and "Wittgenstein, A Symbol of Troubled Times",[2] the latter published in *Philosophy and Phenomenological Research* and the first one in *Realidad;* both articles charged Wittgenstein with liabilities that in the States were in those years easily understood in the light of the blames hold against Russell, first of all the one of atheism. I remember how largely read were on the European Continent a number of papers published by professor Feys in *Revue néoscolastique de philosophie* and subsequently collected in a volume [3] that mingled the views of Russell and Wittgenstein under the denomination of *logical atomism*; such a book was considered by many people in Belgium and in France as the last word on Wittgenstein's *Tractatus* .

I refrain from mentioning the misleading and mistaken sentences that in the fifties were formulated by some of the bosses of Italian Universities and adopted by their claques, all inspired by the obvious presence of Russell's logical elements in the *Tractatus*, first of all the one by Ludwig Geymonat that declared the *Tractatus* to be the work that originated the new philosophical trend that had started from the problems of philosophy of mathematics and had developed into logical positivism, [4] let alone the view of Francesco Barone that considered the *Tractatus* as the

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[1] In *Realidad* 1949, III, pp. 129 – 140.

[2] In *Philosophy and Phenomenological Research* 1953, XIV, pp. 89 – 96. This paper was published also in *Theoria* (Madrid) under the title "Wittgenstein, simbolo de una época angustiada" 1954, II pp. 33 – 38, and reprint in K.T.Fann ed. *Ludwig Wittgenstein : the Man and its Philosophy,* Dell Publ. Co., New York 1967, pp. 107 – 115.

[3] Robert Feys, *Le raisonnement en termes de faits dans la logistique russellienne.* Institut Supérieur de Philosophie, Louvain 1928.

[4] «E' sorta in tali paesi [German speaking countries] *una filosofia a interesse prevalentemente* logico, *che partita dallo studio dei problemi di filosofia delle matematiche, si è ora elevata a questioni di carattere del tutto generale.*[…]. L'opera da cui si può dire prese origine il nuovo indirizzo, è il *Tractatus Logico-Philosophicus* del viennese Ludovico Wittgenstein". L. Geymonat, *Studi per un nuovo razionalismo*, Chiantore, Torino 1945, pp. 3-4.

only work that offers systematically and synthetically the original lines of neopositivism.[5]

However we should keep in mind that in the fifties the community of philosophers at large with the exception of logical positivists still resented little the impact of Wittgenstein. Among the papers presented to the 12th International Congress of Philosophy (12 volumes) not one was dedicated to Wittgenstein's themes and he himself is very rarely mentioned. This state of affairs changed slowly in the sixties and among the papers presented to the 14th International Congress of Philosophy (5 big volumes) only one took Wittgenstein into account [6] with an attention to his mention of the mystical. So, when I published in 1964 my work on Wittgenstein ,[7] in which I tried to give a more balanced account of his views, I met two sorts of reactions. While in Italy it showed in the tightest silence of those that hurried to copy my suggestions without quoting my work, outside Italy there were some perplexities expressed by Strawson,[8] that I suppose were successively dissolved when other manuscripts by Wittgenstein were published and studied. But as such manuscripts and notes and letters were found and published, a larger and larger public realized that there were a lot of ideas that could be picked up here and there and developed without disturbances by

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[5] "Il *Tractatus* del Wittgenstein è l'unica opera che presenti sistematicamente, benché in forma pregnante e sintetica, i tratti originali della concezione neopositivistica". F. Barone, *Il neopositivismo logico*, Ed. di Filosofia, Torino 1953, p. 120.

[6] The papers presented to the XII International Congress of Philosophy in Venice were published in 12 volumes by Sansoni, Florence 1960. I have found mention of Wittgenstein only in vol. 4 p. 235. In the preliminary papers only Ayer mentioned him (*Relazioni introduttive.* Sansoni, Firenze 1960 p. 141). The papers presented to the XIV International Congress of philosophy in Wien (1968) were published in 5 big volumes by Verlag Herder, Wien 1969. In the 3th volume a paper by Leonard H. Ehrlich bears the title "Mysticism of Metaphysics? A Juxtaposition of Wittgenstein, Thomas Aquinas and Jaspers" (vol. 3 pp. 651 – 659).

[7] E. Riverso, *Il pensiero di Ludovico Wittgenstein,* Libreria Scientifica Editrice, Napoli 1964; a new edition, enlarged, appeared by the same publisher in 1970.

[8] P.F. Strawson was nevertheless kind with my book that he reviewed in *Mind*, n. s. LXXV (1965), p. 447.

the austere face of formal logic and by any sort of Occam's razor. In fact in many countries, like France and Italy, what had mostly discouraged the communities of philosophers from paying enough attention to Wittgenstein's *Tractatus*, had been the formal logic that imbibed the propositions of its central core.

Since the fifties I have known some professors of philosophy whose interest in the *Tractatus* was confined in the propositions on mystic, where they found allowances for all sorts of metaphysical and religious flights free from logical shackles and semantic controls. This sort of people have plunged into the posthumously published works by Wittgenstein as in a true paradise of emotional stimulations originated by the very obscurity and difficulty of some of its paragraphs and have indulged to all sorts of swerving speculations depicting Wittgenstein as a mystic, a saint, a gay, a lover of obscurities.

Such people, that can be found mostly among continental philosophers, have ignored that Wittgenstein all along his life was strongly concerned with rigour and clearness of speech and was not ready to indulge or to allow other people to indulge on his behalf to ungrounded judgement or to ambiguous and fantastic verbalizations. So it will be always useful to remember that Wittgenstein was basically a logically oriented mind, caring rigour and conceptual neatness, and this tendency came to him from his early studies in engineering, mathematics and formal logic that Russell had taught him.

2. Russell's pure formal logic.

The whole architecture of *Tractatus* rests on the achievements of Bertrand Russell's logical commitment that had brought him to the discovery of the fittest symbolic instruments to insulate logical forms. We know that the existence of logical form embedded in the speech was known by Aristotle and his commentators, to Scholastic philosophers of Middle Ages and

Leibniz and that Peano and Frege had contributed very important clarifications to the understanding of logical structures of mathematics and common language. But it was the work of Bertrand Russell that produced the most convenient technical instruments to represent strict formalities of logic and to allow all sorts of logical operations.

Such a technical achievement afforded the means of conceiving of logic as a set of expressions completely devoid of content and able to absorb all branches of non applied mathematics in a way that granted Russell the right to affirm that formal logic is the same thing as pure mathematics and " pure mathematics consists entirely of assertions to the effect that, if such and such a proposition is true of *anything*, then such and such another proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is, of which it is supposed to be true. Both these points would belong to applied mathematics […]. Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true".[9] When Russell wrote these lines in 1901, he was

[9] B.Russell, *Recent Works on the Principles of Mathematics* (1901), in *Collected Papers of Bertrand Russell*, vol. 3, Routledge, London and New York 1993, p.367. This paper is very important for understanding the development of Russell's logical thought; it was first published in *International Monthly* (1901, IV pp.83 – 101), July issue, and written in January of the same year. It was enthusiastically received by Couturat and by T.S.Eliot (*Nation* 1918, p. 768) and criticized by Jourdain (*Scientia* 1919, p. 63). Under the title *Mathematics and the Metaphysicians* it was reprinted in *Philosophical Essays* (1910), that became *Mysticism and Logic* in 1917 (see the ed. of 1963 by Allen and Unwin, London pp. 59 – 74). In this paper, that was destined to a popular audience and therefore indulges in striking sentences, Russell espressed for the first time the view that pure mathematics is identical with logic. In a note on p. 62 of the reprint in *Mysticism and Logic*, he declared that when the paper was written in 1901, he did not know the writings of Frege. It will be of some interest to know that in 1909 in Italy Benedetto Croce (in *Logica come scienza del concetto puro*, Laterza , Bari, rist. 1967 p. 214) quoted the view of Russell that in mathematics we never know what we are talking about, nor whether what we are saying is true (without mentioning Russell and his paper) as evidence that mathematics is not a science (see E. Riverso "Il linguaggio nella logica come scienza del concetto puro" in *Riscontri* (Avellino) 1988, X , pp. 49 – 79).

strongly concerned with the concept of *any* [10] that he identified with the concept of *variable*, a concept essential to his way of insulating pure logic from all contents or applications. In fact the draft of P*rinciples* he wrote in 1901 contained a remarkable effort to clarify the logical concept of variable beyond the results achieved by Peano.[11] In this draft Russell introduced also the definition of pure mathematics as the class of all propositions of the form "*a* implies *b*", where *a* and *b* are propositions each containing at least one variable and containing no constants except logical constants or such as can be defined in terms of logical constants.[12]

He affirmed that "when once apparatus of logic has been accepted, all mathematics necessarily follows. And logic may be defined as 1) the study of what can be said of *everything*, i.e. of the propositions which hold of all entities, together with 2) the study of the constants which occur in true propositions concerning everything".[13]

Russell made profit of Peano's and later of Frege's technicalities touching functions and variables, but he exploited the analysis of all sorts of propositions into functions and arguments in such a way as to abstract the pure logical form and make it treatable in all sorts of logical operations without any danger of confusion with possible contents. So he showed for the first time how pure formal logic was to be conceived of, how the

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[10]He had expressed this concern in a letter to G. E. Moore (August 16, 1900) after he had met Peano during the Congress of Philosophy of Paris of 1900. See B. Russell, *Collected Papers*, vol. 3 p. 181.

[11] See B. Russell , *Collected Papers*, vol. 3, pp. 185 – 208.

[12] Ibidem p. 185. Subsequently Russell made a distinction between *free* or *real variable* and *bound* or *apparent variable.* The first one is a variable without quantification, as in "($f\,x$) implies ($y\,x$)". The latter is quantified, as in "(x). (fx) implies (yx)" or "($\exists\,x$). (fx) implies (yx)". We should understand that in the reported definition of mathematics Russell required that *a* and *b* contained at least one free real variable. See *Principia mathematica*, vol. I (Cambridge University Press, Cambridge 1910) pp. 127 – 186. This section was substituted in the second ed. of *Principia* (1925) by ch. 8 in append. A, ibidem vol. I pp. 635 – 649.

[13] Ibidem p. 187.

whole mathematics can be deduced constructively from a little number of primitive propositions through substitution and simplification [14] and how formal logic ant its mathematical development can be applied to reality through the substitution of values to variables. Unfortunately his logical technology was flawed by the theory of types invented to protect it against paradoxes and such a theory was bound to be criticized as inconsistent with strict formal logic,[15] but without the sort of formal logic invented by Russell, who exploited technicalities and concepts invented by Peano and Frege, Wittgenstein would not have been in the condition of producing the thoughts he expounded in *Tractatus*.[16] Obviously he did not omit relevant criticisms touching some important points of that logic as the theory of types and disposed of paradoxes of reflexivity on the ground that the sign for a function cannot contain itself as its argument and therefore a paradox of this sort simply cannot be constructed.[17]

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[14] The development of more complex propositions (therefore the ones of mathematics) from the simplest ones was subsequently illustrated by Russell as a constructivist deduction that employed simplification and substitution. Simplification is permitted by inference: a proposition *p* is asserted, and a proposition "*p* implies *q*" is asserted, so *q* is asserted and the assertion of *p* can be omitted: "|— *p* . (*p* implies *q*)" becomes "|— *q*" (*Principia Mathematica* I, pp. 8 – 9). Substitution is effected by writing a determined term in all mentions of another term in a proposition; this operation is represented by an horizontal line as in a ratio: if *p* should substitute *q*, we write *p* above the line and *q* underneath (*Principia Mathematica* I, pp. 98 – 108).

[15] See F.P.Ramsey, *The Foundations of Mathematics and other Logical Essays*, Routledge and Kegan Paul, London 1931 pp.63 – 65. Nevertheless it is probably possible to formulate the theory of types without mentioning meanings of signs. The theory of logical types was illustrated by Russell in a paper in French (in *Revue de métaphysique et de morale*, XVIII, 1910 pp. 263 – 301) and in English (see B. Russell, *Collected Papers*, vol. 6, Routledge, London 1992 pp. 4 – 31), and in chapters 2 and 3 of the "Introduction" to *Principia Mathematica* (vol. I, 1910 pp. 37 – 84).

[16] *Tractatus logico-philosophicus*, Routledge and Kegan Paul, London 1961, 3.331 – 3.333.

[17] Ibidem 3.331: "Der Irrtum Russells zeigt sich darin, dass er bei der Aufstellung der Zeigenregeln von Bedeutung der Zeichen reden musste".

3. No logical constants

It is easy to realize that Wittgenstein had adopted Russell's quest of strictly formal logic and was concerned with improving what Russell had attained in this quest. This adoption was a consequence of Wittgenstein being a pupil of Russell, a member of Trinity College in Cambridge, and an Advanced Student admitted to a Course of Research under the direction and supervision of Russell [18] during three terms of 1912 and two of 1913. In the spring of 1913 he went to Norway to develop freely the input received by Russell. In April 1914 he went to Austria to enrol in the army at the outbreak of war. So he did not attend any

[18] A letter of June 5, 1912 by J.N. Keynes, University Registrary, to Dr W. M. Fletcher tutor at Trinity College (Cambridge) asked "Mr. Bertrand Russell to be kind enough to act as the Director and Supervisor of the Student". See L. Wittgenstein, *Letters to Russell, Keynes and Moore*, B. Blackwell, Oxford 1974 p. 1. Russell had accepted an appointment to lecture in Cambridge, at Trinity College, on logic and philosophy of mathematics to begin in October 1910 for five years. He took some rooms in Nevile's Court and separated also physically from his first wife Alys, having their marriage terminated years before. Wittgenstein was a student in the University of Manchester in the autumn of 1911 but on October 18 of that year he went to Cambridge, where he met Russell and probably appreciated his views on logic and mathematics at the point that he resolved to become his pupil. In fact on February 1, 1912 he became a member of Trinity College in Cambridge and resided there for three terms of 1912 and the first two of 1913. It was in this period that he absorbed directly from the teaching of Russell and from discussions he had with him his views on formal logic and immediately started with his own reflections on them that produced the propositions of *Tractatus*. This first encounter of Russell with Wittgenstein was recounted by him in his correspondence to Lady Ottoline, when he was going to begin his second year of teaching. He wrote that he was discussing with Ogden: "when an unknown German appeared, speaking very little English but refusing to speak German. He turned out to be a man who had learned engineering at Charlottenburg, but during his course had acquired, by himself, a passion for the philosophy of mathematics, and has now come to Cambridge on purpose to hear me" (the passage of the letter to Lady Ottoline is in B. Russell, *Collected Papers* vol. 6, 1992, p. XXXIV).

more the teaching of Russell. It is doubtful if he read anything of *Principia Mathematica* that were published in those years.[19]

We know that in the years in which he had achieved the symbolic technicalities of his system of formal logic, Russell adopted a sort of Platonism that made relations and formalities into a sort of rational objects endowed of eternity and immutability, the target of top knowledge attainable by men [20] and the specific topic reserved to philosophy; so he identified logic with philosophy as both had to study such rational objects.[21]

According to my view this sort of Platonism was of the utmost importance in the working out of formal logic by Russell, though it does not enter in any way its symbolic constitution, what allowed him to throw it off in 1914 and try to keep safe the symbolic system as a linguistic structure.[22] Unfortunately I have remarked that Russell's Platonism has been scarcely noticed by

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[19] Bertrand Russell had written *Principia Mathematica* with Alfred North Whitehead, but *the* Introduction and the system of ideas and technicalities that supported the work were his own (see E. Riverso, *Il pensiero di Bertrand Russell*, Libreria Scientifica Editrice, Napoli 1972 III ed. pp. 57 – 185. It can be useful to see the first edition of this work, 1958 Istituto Editoriale del Mezzogiorno, that is rich in detailed informations). The first edition of *Principia Mathematica* was published in Cambridge at the University Press: 1910 I vol., 1912 II vol., 1913 III vol.

[20] The best summary of this Platonism can be found in *The Problems of Philosophy* that Russell published in 1912 at Home University Library (reprint by Oxford University Press 1959).

[21] In 1914 Russell told and wrote that "philosophy is the science of the possible […]. Philosophy, if what has been said is correct, becomes indistinguishable from logic. […] The essence of philosophy as thus conceived is analysis, not synthesis" ("On Scientific Method in Philosophy" in *Collected Papers* vol. 8, 1986, pp. 57 – 73, see pp. 65 – 66). In the same year he delivered (March and April) the Lowell Lectures in Boston and had them published by the Open Court Publishing Co. under the title *Our Knowledge of the External World*; the second of these lectures developed the view that logic is the essence of philosophy.

[22] In the "Introduction" to the second edition of the *Principles of Mathematics* (Allen and Unwin, London 1937, p. XI) Russell declared: "Logical constants, therefore, if we are to be able to say anything definite about them, must be treated as part of the language, not as part of what language speaks about. In this way, logic becomes much more linguistic than I believed it to be at the time when I wrote the *Principles*".

students and I myself have had an argument with Sir Alfred on this topic.[23] But we should keep in mind that Russell's logical teaching imparted to Wittgenstein in 1912-1913 was supported by a Platonism that made logic into the knowledge of a system of objects that Wittgenstein , trained as an engineer, could not admit of. Logical constants are among these objects and while Russell passionately defended their existence, Wittgenstein rejected them not only by objecting during his lessons and discussions [24] but also by letters written to him.[25] So the logic of Russell that passed into *Tractatus* was wholly devoid of the Platonic or realistic [26] support it had had in Russell's view. While the world of Russell was composed of two sorts of things, universals (relations or logical constants) and particulars (sense-data), the world of *Tractatus* was the totality of facts (*Tatsachen*), that is of the extant states of affairs (*das Bestehen von Sachverhalten*) [27] and nothing more.

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[23] See E. Riverso, "Ayer's Treatment of Russell" in L.E. Hahn ed. *The Philosophy of A. J. Ayer*, Open Court, La Salle (Ill.) 1992 pp. 517 – 541, with the reply by Ayer pp. 542 – 544; the reply to Ayer by Riverso was in "Wittgenstein versus Russell: On Logical Realism" in *Wittgenstein Studies*, Passau, Deutsche Ludwig Wittgenstein Gesellschaft e. v., Disk 2,1995.

[24] In his correspondence with Lady Ottoline of 1911 – 1912 many references to Wittgenstein as his new pupil show how hard Russell felt to face the difficulty of Wittgenstein to accept the logical objects that crowded the pure formal logic he had built up. At first Russell thought Wittgenstein to be a German that was "much interested" by him, but the next day he declared: "My German friend threatens to be an infliction - he came back with me after my lecture and argued till dinner-time – obstinate and perverse, but I think not stupid". See B. Russell, *Collected Papers*, vol. 6 pp. XXX – XXXIX.

[25] In a letter to Russell of 22/6/1912 Wittgenstein wrote: "The prop[osition]s of logic contain ONLY APPARENT variables and whatever may turn out to be the proper explanation of apparent variables, its consequence *must* be that there are NO *logical* constants". L. Wittgenstein, *Letters to Russell, Keynes and Moore*, B. Blackwell, Oxford 1974, p. 10.

[26] Since 1911 Russell had openly supported a realism which admitted of two sorts of objects as independent of mind or consciousness: particulars (sense-data) and universals (relations). See his paper "Le réalisme analytique" in *Bulletin de la société française de philosophie* 1911, XI pp. 53 – 82, reprint in *Collected Papers* vol 6 pp. 410 – 432.

[27] L. Wittgenstein, *Tractatus* 1.1 and 2.

4 . Evolution of logical symbolism

Wittgenstein's contention that there are no logical constants was an attack upon Russell's realism of universals or relations because Russell included logical constants and also prepositional functions among relations, and conceived of them as the pieces of a world possessing "not only truth , but supreme beauty - a beauty cold and austere , like that of sculpture" .[28]

Russell strongly resented this attack [29] but, when it came, he was going to face a profound mental crisis partly due to the outburst of First World War. So in the late 1914 he was no more in the mood of defending Platonism and realism of universals. However the views that Russell could formulate after 1913 to resist or to yield to Wittgenstein had no possibility of impact on this one, because Wittgenstein, as I have said, wanted to think by himself and in October 1913 went to Norway to be quite alone in his work.

Russell's logic that was going to be modified by Wittgenstein and adopted in the *Tractatus*, was the product of a ten years work partly effected with the help of Whitehead and supported by a passionate belief in a world of subsistent realities of pure rational nature represented on the paper by symbols of logistics.

Russell had been strongly impressed by the researches of Peano that had given the symbol of implication a central position in the analysis of mathematical concepts.[30] So he had adopted it,

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28 B. Russell, "The Study of Mathematics" publ. As ch. IV In *Mysticism and Logic*, Unwin Books, London 1917 p. 49.

29 "I thought of mathematics with reverence, and suffered when Wittgenstein led me to regard it as nothing but tautologies", B. Russell, "My mental Development" in P.A.- Schilpp ed. *The Philosophy of Bertrand Russell*, Tudor, New York 1951 p. 19.

30 The symbol for implication as a converted C was introduced into logical usage by Peano as meaning *tunc, seque, si … tunc* (from *latino sine flexione*) in

probably with the implicit feeling that it could represent a fundamental tie in the world of logical realities, and outlined in 1906 a theory of implication [31] that was a first sketch of his project of building mathematics by logical elements. It was only the need of reducing the number of logical constants that induced him to adopt the definition of implication in terms of negation and disjunction [32] and Russell agreed that "theoretically, all definitions are superfluous".[33] But it is impossible to deny that when H.M.Sheffer showed that all functions of propositions can be defined by the function of double negation ,[34] implication was deprived of all possibility of being felt as an objective tie between propositions.[35]

Formulario mathematico (1908, see ed. Cremonese, Roma 1960 p. 3) and largely used in the whole text. Russell adopted it according to a more restricted and uniform usage; see G.H. Moore "Introduction" to vol. 3 of B. Russell, *Collected Papers*, pp. XLV-XLVII. For the *Formulario* of Peano see H.C. Kennedy, *Peano. Life and Works of Giuseppe Peano*, Reidel, Dordrecht 1980 pp. 44- 50, 118 – 124.

[31] "The Theory of Implication" in *American Journal of Mathematics* 1906, XXVIII, pp.159 – 202. See also: "'If' and 'Imply'". In *Mind* 1908 n.s. XVII p. 300.

[32] "*p* implies *q*" can be defined also by negation and conjunction: *p* implies *q* .= . ~ (*p* · ~ *q*). But, as also conjunction can be defined by negation and disjunction, the technical need to reduce the number of functions of propositions rendered preferable the assumption of negation and disjunction as the two fundamental logical constants to which all others could be reduced. See *Principia Mathematica*, vol. I pp. 7, 12.

[33] Ibidem p. 11.

[34] H. M. Sheffer, "Total Determinations of Deductive Systems with Special Reference to the Algebra of Logic" in *Bulletin of the American Mathematical Society*, vol. XVI, and "A set of five independent postulates for Boolean algebra, with application to logical constants" in *Transactions of the American Mathematical Society* 1913, XIV pp. 481 – 488.

[35] Russell adopted the functor of double negation (the *stroke functor*) in the second edition of *Principia Mathematica* of 1925 (see I vol. "Introduction" pp. XIII, XV – XIX, XXV – XXVIII, 659 – 666) and praised Sheffer for his discovery (what was instrumental for Sheffer's progress in academic career), but afterward it was found that this functor had been yet discovered by Peirce and the paper by Peirce had passed unnoticed (see C. S. Peirce "A Boolean algebra with one constant" in C. S. Peirce *Collected Papers*, C. Hartshorne and P. Weiss, edd. Cambridge (Mass.) 1933- 1934, vol. IV pp. 13-18).

This could no more trouble Russell after 1914, but Wittgenstein succeeded in publishing the *Tractatus* in 1921,[36] when he was acquainted with stroke functor [37] and thought that this functor can be useful to show the inner connexion of logical formulas.[38] He realized that every truth-function can be conceived of as the result of one or more successive applications to elementary propositions of the operation of negation [39] and assumed that negation of a proposition means that it is false or that it is the opposite of true. Consequently, as Wittgenstein introduced (almost in the same time as Post) the technical possibility of calculating truth and falseness of all truth-functions starting from truth or falseness of elementary propositions, through the introduction of schemata of representation of truth-possibilities,[40] falseness and truth became mere operative signs whose meaning could be wholly ignored, what opened the way to strict formalism and to forsaking Russell's concept of truth,[41] that had been very rich in philosophical contents and a fundamental support of his logical view.

In *Tractatus* the absence of logical constants, the transformation of the theory of logical types into a rule of usage of symbols (no symbol of function can appear both as argument and function in the same formula) [42] and the technical calculability of truth-value of truth-functions made the world into

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[36] In *Annalen der Naturphilosophie*.

[37] See *Tractatus* 5.1311. Remark that the general form of an operation given in *Tractatus* 6, 6.001, 6.01 opens the way to adoption of the *stroke function*.

[38] In *Prototractatus*, that was probably finished in Summer 1918, nothing appears that resembles the *stroke function*; see *Prototractatus, an early version of Tractatus Logico-Philosophicus by Ludwig Wittgenstein* ed. by B. F. McGuinness and others, Routledge and Kegan Paul, London 1971.

[39] See *Tractatus* 5.5, 5. 501, 5. 502, 5. 51, 5. 512, 5.52, 6 – 6. 01.

[40] See *Tractatus* 4. 31, 4. 441, 4.442, 5.101, etc.

[41] Russell had published in the *Proceedings of the Aristotelian Society* (1907) a paper under the title "On the Nature of Truth". The first two parts of this paper were reprinted in a collection of essays: *Philosophical* (1910, Longmans, London). The complete manuscript has been published in *Collected Papers*, vol. 6, under the title " On the Nature of Truth and Falsehood".

[42] See *Tractatus* 3. 331, 3.332

a totality of mere combinations of objects with no logical glue or supporting structure, because the logical frame of a proposition was given by the mere distribution of objects and not by something added to them in the role of logical constants.

The need of analysing the world into *Sachlagen* and these into ultimate simple objects dominates the *Tractatus*. It is difficult to abstain from finding a dependence of this need from Russell's theory of descriptions,[43] though no mention of it appears in the whole *Tractatus*. Russell's theory of descriptions dissolved presumptive objects built up by description, that is by formulas of the form "the so-and-so", into complex propositions that are functions of elementary propositions. In *Tractatus* elementary propositions is the simplest kind of proposition and asserts a state of affairs.[44] As an elementary proposition consists of names and is a nexus of names,[45] and as names are the simple symbols [46] that represent objects in the proposition [47] so that propositions are functions of names [48] and "the analysis of propositions must bring us to elementary propositions which consist of names in immediate combination",[49] Russell's theory of descriptions is necessary to obtain this analysis that brings us to real objects denoted by names.

5. The principle of extensionality

[43] Russell's *theory of descriptions* was first formulated in the well known paper "On Denoting" in *Mind* (1905, n.s. XIV pp. 479 – 493), reprint in B. Russell, *Logic and Knowledge* ed. by R.Ch. Marsh (Allen and Unwin, London 1973 pp. 103 – 119) and in *Collected Papers* (vol. 4, Routledge, London and New York 1994, pp. 415 – 427). It entered Russell's system of formal logic in *Principia* (vol. I pp. 30- 36) and was popularly expounded in Russell's *Introduction to Mathematical Philosophy*, Allen and Unwin, London 1919 pp. 167 – 180.

[44] *Tractatus* 4.21.

[45] *Tractatus* 4. 22.

[46] *Tractatus* 4. 24.

[47] *Tractatus* 4. 1272.

[48] *Tractatus* 4. 24.

[49] *Tractatus* 4. 221.

When engaged in working out of his logic, Russell made a fundamental distinction between propositions generated by a function and one or more arguments [50] and propositions generated by logical constants and one or more propositions. This distinction is suggested since the beginning of the draft of 1901 of *Principles of Mathematics*, where pure mathematics is defined as " the class of all propositions of the form '*a* implies *b*', where *a* and *b* are propositions each containing no constants except logical constants or such as can be defined in terms of logical constants".[51]

This means that propositions of mathematics (and logic) are not propositions on objects but propositions on (possible) propositions (propositional functions) that have no apparent variables and therefore are called *elementary propositions*.[52] The distinction between elementary and complex propositions became more and more neat in Russell's thought and was of the utmost importance in the logical system of *Principia*. Wittgenstein was strongly impressed, probably since the time he attended the lessons of Russell in Cambridge (1912 – 1913), by the possibility of reserving elementary propositions to represent individual objects with their relations while assuming complex propositions as logical compounds of elementary propositions.

The truth or falseness (truth-value) of elementary propositions could be logically reduced to few logical structures (implication, disjunction, negation, logical product, equivalence; all definable by negation and disjunction). The only sort of

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[50] Functions with two or more arguments are relations.

[51] B. Russell, *Collected Papers* vol. 3, p. 185. In the final draft we read: "Pure Mathematics is the class of all propositions of the form '*p* implies *q*', where *p* and *q* are propositions containing one or more variables, the same in the two propositions, and neither *p* nor *q* contains any constants except logical constants" (B. Russell, *The Principles of Mathematics*, Allen and Unwin, London 1903 p. 3). Here the symbols *p* and *q* are definitely adopted to represent propositions (the symbols *a*, *b*, *c*, … are adopted to represent values of the variables of a propositional function), but in the second edition of *Principia p* in some sections is substituted by *fx*, what makes explicit that it is the case of propositions about individuals (*Principia Mathematica* 1925 vol. I p. XIV).

[52] See B.Russell, *Introduction to Mathematical Philosophy* p. 161.

propositions that had resisted to this reduction were the ones of the form "*A* believes *p*", whose truth-value is not dependent on the truth-value of *p*. He had invented the so-called "*multiple theory of judgement*" [53] that transformed such propositions into propositions enouncing a relation among *A* and the constituents of *p*; consequently such propositions became elementary and *p* ceased to be an autonomous proposition with a truth or falseness of its own.

Wittgenstein's adoption of Russell's distinction between elementary and complex propositions and of dependence of truth-value of the latter on truth-value of the first brought him to the formulation of the so called *thesis of extensionality*,[54] that gave much technical rigour to the logic of propositions built up by Russell. The *thesis of extensionality* states that all propositions are either elementary propositions or truth-functions of elementary propositions and, as Wittgenstein regarded elementary propositions as truth-functions of themselves, he could assert that any sentence is a truth-function.[55]

This thesis was developed into schemata (or matrices) where it was made clear how the possible truth-values of each sort of complex proposition (implication, negation, alternation,…) is dependent on the values of elementary propositions whose the complex proposition is a function. This was a very important improvement of Russell's logical technicalities, that Russell himself adopted.[56]

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53 See *Principia* vol. I pp. 43 – 44. For a criticism of this view see P. Geach, *Mental Actes*, Routledge and Kegan Paul, London 1957 pp. 45 – 60.

54 The name 'thesis of extensionality' was invented by Rudolf Carnap in *The Logical Syntax of Language*, Routledge and Kegan Paul, London 1937 pp. 245 – 247.

55 The most typical propositions of *Tractatus* where the thesis of extensionality is adopted and formulated are : 4 . 41, 4. 51, 4. 52, 5, 5. 01, 5. 234, 5. 3, 5, 54, 6. 001.

56 In the "Introduction" to the second edition of *Principia Mathematica* (vol. I pp. XV – XXXIX) Russell employed the names 'atomic proposition' and 'molecular propositions' to refer to elementary and complex propositions. See also: B. Russell, *An Inquiry into Meaning and Truth*, Allen and Unwin, London 1940 p. 211.

But the propositions of the form "*A* believes *p*" received a different treatment by Wittgenstein[57] that consisted in resolving it into a logical product: "*A* utters *s* and *s* says that *p*", where *s* is the verbal presentation of *p*. The adoption of the *thesis of extensionality* in *Tractatus* is fundamental for the whole development of the book.[58] It supports the doctrine of facts and objects, the doctrine that an elementary proposition is a picture of reality (of a *Sachverhalt*) showing its real logical form,[59] the idea of reality divided into facts wholly independent of each other,[60] the *picture theory*.[61]

In fact the system of logic elaborated by Bertrand Russell in a ten years toil, was corrected and modified by young Wittgenstein, who made it into such pure formalities [62] that cannot be spoken of but only shown. So it became the suggestion of a new ontology on which a new foundation of scientific description of reality should be possible through the adoption of a rigorous language and the forsaking of all sorts of slipshod reasoning.[63]

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[57] *Tractatus* 5.541, 5. 542, 5. 5422, 5. 5423, 5.55, 5.551.

[58] See D. Favrholdt, *An Interpretation and Critique of Wittgenstein's Tractatus,* Munksgaard, Copenhagen 1965, pp. 9 – 28.

[59] See *Tractatus* 4. 01. Wittgenstein acknowledged that it was Russell who performed the service of showing that the apparent logical form of a proposition need not to be its real one (see: 4. 0031). Here an implicit commendation of Russell's theory of descriptions is probably present.

[60] "Die Sachverhalte sind von einander unabhängig", *Tractatus* 2. 061. The reciprocal independence of *Sachverhalten* produces the reciprocal independence of elementary propositions, what permits their treatment according to schemata of complex propositions.

[61] According to Wittgenstein elementary propositions are pictures of states of affairs, but complex propositions are mere logical compounds devoid of any frame existing in the world. See James Griffin, *Wittgenstein's Logical Atomism,* Oxford University Press, London 1964 pp. 87 – 111.

[62] See *Tractatus* 3. 262, 4. 022, 4. 0621, 4. 121, etc.

[63] See E. Riverso, *Il pensiero di Ludovico Wittgenstein*, Libreria Scientifica Editrice, Napoli 1970 (I ed. 1964) pp. 49 – 180.