Model Validation and Test Portfolios in Financial Regulation

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Abstract. We describe the problem of test portfolio selection and construction that arises in the context of regulators' use of test portfolios to validate and benchmark the models that financial firms use to determine their regulatory capital requirements. We provide an example and discuss whether Mechanised Reasoning might be used to help provide a more structured approach to this problem.

1 INTRODUCTION

Regulators allow the use of financial firms' own ("internal") models for calculation of regulatory capital requirements for several risk types and business areas – the focus of this note is on market risk in the trading book, i.e. fluctuations in value of assets held with "trading intent". Such assets are required to be re-valued ("marked to market") daily, it is therefore implicitly assumed that a reliable market price is available every business day, and market risk is broadly defined as the uncertainty in the future value of the product due to changes in market prices.

Regulated Firms with trading books are required to hold capital against this risk, to absorb "unexpected" losses due to market moves, and under the under the so-called Basel accords are allowed to use their own ("internal") models to calculate this capital requirement, subject to regulatory approval. This option is mainly used by large investment banks, and the markets and investment banking divisions of large financial groups.

Regulators' model approval approach typically relies on firms' internal model validation and control functions, but this is increasingly being supplemented by hypothetical portfolio exercises (where several firms are given the same portfolios to evaluate), as concerns increase over variability of capital requirements and adequacy of risk capture following the experience of the most recent financial crisis. The importance of test portfolios will increase, as regulators are becoming more interested in the cross-industry benchmarking of models and capital – this is very likely to be reflected in future versions of regulatory requirements.

2 MODELS

The term "model" is used in two distinct ways in the present context: "valuation" models and "capital" models.

Valuation models are used to calculate the value of financial products given a state of the market – note that the values of complex financial instruments are calculated as functions of the (more readily observable) values of simpler instruments, also called "risk factors", for example currency exchange rates.

Capital models are used to calculate capital requirements, and are based on estimating a future distribution of market values (or equivalently, changes in market values from the current value, or "profit & loss") over a given time horizon, and calculating a metric (VaR, Expected Shortfall) from this distribution.

Capital models calculate future distributions of product valuations by first modelling a joint distribution of future values of risk factors, and then using appropriate valuation models for each product, taking the modelled risk factor distribution as input.

3 MODEL VALIDATION

For the validation of capital models, the set of financial products to which the model applies is assumed to be known; the values of these products are in turn dependent on a set of known risk factors.

Test portfolios for the model will therefore be defined either by subsets of the set of products in scope, or by subsets of the set of all risk factors, which will in turn define test portfolios of (potentially simplified) products.

As a simple example, consider a product scope of interest rate swaps and FX forwards. The associated risk factors are the FX spot rate and the interest rate curve, the latter represented by a set of interest rates at different maturities. A test portfolio for this scope could consist of some combination of swaps and FX forwards, or, when analysed in terms of risk factors, might consist of portfolios of FX forwards and zero coupon bonds, the latter having exposure to just one point on the curve.

The validation of capital models can usefully be thought of as having three distinct (although obviously related) objectives.

First, we need to verify that risk factors for individual traded products are appropriately defined, and that the valuation model used for the product *in the capital model* is sufficiently accurate. There are typically two issues in this regard: one, not all risk factors for the product are included in the capital model, and two, the capital model could be using an approximation of the product's valuation function (for efficiency and infrastructure reasons Taylor series approximations are often used). This aspect is typically verified by comparing the actual historical profit and loss (P&L) series for the product with the corresponding P&L calculated by the capital model, i.e. via a form of back testing.

Second, we need to verify that risk factor data used in the capital model is complete and sufficiently granular, i.e. risks that are similar but not the same should be recognised by the model. This aspect is typically verified through test portfolios consisting of simple strategies that give exposure to the risk factors being tested but have little or no exposure to other risk factors. The capital model should not give a zero capital requirement for such a strategy – this would indicate that the model does not distinguish between these risk factors, and therefore misses "basis risk".

Third, the impact of risk factor covariance and distributional assumptions should be tested, to establish whether the model will give sufficiently conservative capital levels for all plausible product combinations (portfolios) and market conditions (normal, stressed). This is determined by properties of the capital model, including the type of metric (e.g. VaR, Expected Shortfall), the amount of data used to calibrate the model, the framework (Historical, Monte Carlo, weighted or un-weighted).

The main test used for this aspect is – again – back testing of capital against historical P&L time series for test portfolios, ideally compared across different model types. The additional dimension in this instance is of course test portfolio selection.

4 EXAMPLES

Products and strategies run by a typical Credit Trading business can be used to illustrate the three aspects of model validation discussed above.

A "high-yield" corporate bond has a low credit rating and therefore a high probability of default. An appropriate valuation model will recognise the fact that, for large moves in the bond's credit spread, corresponding price moves should be limited so that the position cannot lose more than the assumed loss given default indicates. A capital model using only the credit spread delta ("CS01") to price the bond will not measure the risk of the position correctly, as it will linearly extrapolate price moves from spread moves.

A "negative basis trade" is a strategy where the trader owns a bond, and also buys protection on it via CDS, when the CDS spread is low relative to the bond's own credit spread. Many capital models use the CDS spread as a proxy for both the CDS and bond spreads, and will therefore not measure the risk of this strategy correctly.

The capital calculated for a portfolio of CDS contracts on highly correlated names will be highly sensitive to both small changes in position and small changes in risk factors (credit spreads), potentially giving an unstable capital measure.

5 CONCLUSIONS & FUTURE WORK

The crux of the test portfolio problem is that, while back testing individual products and strategies across different models and time periods arguably provide most of the information required to validate pricing and risk factor selection for the purpose of capital modelling, the "portfolio effect" means that not all portfolios are equally well capitalised at all times in all models.

This effect is more pronounced during periods of stress, as the usual relationships between products (and asset classes) break down.

All three aspects discussed in Section 3 are (or should be) individually validated by firms, using a range of approaches including test portfolios. Regulatory test portfolios necessarily need to test all three, interrelated, aspects. The problem is to design a set of portfolios that does this efficiently, and is further complicated by the fact that information on valuation models and data is typically limited.

There is also a significant additional problem: regulators typically cannot directly verify the correctness of the actual implementation of a firm's capital model, i.e. whether it is implemented according to documented specifications in software and hardware.

Given the particular requirements of regulatory test portfolios, we think that Mechanised Reasoning could provide model validators with new procedures to discover efficient test portfolios beyond traditional portfolio replication and optimisations techniques, particularly given that regulatory test portfolios will have a range of success criteria, or that the determination of success criteria might even be part of the test. This contrasts with other techniques where a desired level of risk, return, or exposure is typically first specified.

The Test dataset provides the gold standard used to evaluate the model. It is only used once a model is completely trained(using the train and validation sets).Â Note on Cross Validation: Many a times, people first split their dataset into 2 â€" Train and Test. After this, they keep aside the Test set, and randomly choose X% of their Train dataset to be the actual Train set and the remaining (100-X)% to be the Validation set, where X is a fixed number(say 80%), the model is then iteratively trained and validated on these different sets. There are multiple ways to do this, and is commonly known as Cross Validation.